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NETWORK SCIENCE STUDIES

Specific statistical properties of the strength of links and nodes of the Estonian network of payments

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Abstract. We investigated the strength of the interactions of the elements of the Estonian network of payments (link weight of payments and volume of payments) by the realization of particular experiments. Specific statistical measures of this network, which combine the topology of the relations of the strength of links and nodes and their specific weights, were studied with the purpose of discovering beyond the topological architecture of our network and revealing aspects of its complex structure. Moreover, scale-free properties between the strengths and the degree values were found. We also identified clear patterns of structural changes in such a network over the analysed period.

Key words: economic networks, complex systems, scale-free networks, weighted networks, strength of nodes.

1. INTRODUCTION

Complex network systems have been studied across many fields of science [1–4]. Undoubtedly, many ystems in nature can be modelled as networks where the elements of the nodes are the elements of the system and the links represent the interactions between these elements. Some examples of such systems are technological networks such as the internet [5] (a network of routers or domains connected via cables), the World Wide Web [6] (where nodes are HTML documents connected by links pointing from one page to another), power grids [7] (electricity networks), social networks [8,9] (such as acquaintance networks and collaboration networks), biological and metabolic networks [10,11], transport networks [12] (worldwide airport network), and economic networks [13] (Japanese bank transaction network). In the last decades, networks have received a great deal of attention and a great portion of recent research has focused on statistical and topological properties, for example, the small-world property [14] and scale-free behaviours [15].

Most of the real networks, alongside with their complex topological structure, display a gradation of interaction regarding connections and their intensities; this is commonly quantified by the link weight. The link weight reveals significant functional properties such as the concentration of friendships between people in social networks, the quantities of flows of money between banks, and the capacity to pass information in a network of communication or transport.

The research questions of this study are the following: Which is the characterization of the strength of the links of the nodes of the Estonian network of payments? Is there any relevant relationship between the weighted quantities and the underlying network structure?

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In this study, we characterize the links by investigating the strength of the interactions of the elements of our network (link weight of payments and volume of payments). We analyse specific statistical measures of the weighted Estonian network of payments that combine the topology of the relations of the strength of links and nodes and their specific weights with the purpose of discovering beyond the topological architecture of the network and revealing aspects of its complex structure.

Section 1 provides a general introduction and an overview of the objectives. In Section 2, we deliver a description of the data set used in this study. Section 3 provides a literature review of related studies. Section 4 presents the methods used to develop this study, while Section 5 gives our main results. Section 6 concludes with a discussion of our results.

2. DATA

Our data set was obtained from the databases of Swedbank, one of the leading banks in the Nordic and Baltic regions of Europe operating actively in Estonia, Latvia, Lithuania, and Sweden. As all the information related to the identities of the nodes is very sensitive, it will remain confidential and cannot be disclosed. Since ~80% of Estonia's bank transactions are executed through the Swedbank's system of payments, the data set is unique in its kind and very interesting. Hence it reproduces fairly well the trends of flows of money of the whole Estonian economy, and we use this dataset as a proxy of the economy of Estonia.

A network is a set of nodes connected by links. In this study, the nodes represent companies and the links represent the payments between the companies. The network of payments is defined by three matrices that map the whole image of the network: A is an undirected connectivity symmetric adjacency matrix $A_{N\times N}$, where N is the total number of nodes in the network and two nodes have a link if they share one or more payments; then each element represents a link as follows: $A^u_{ij} = a^u_{ji}$, where $A^u_{ij} = 1$ if there is a transaction between companies i and j or $A^u_{ij} = 0$ if there is no transaction between companies i and j. The weighted connectivity matrix B contains the number of transactions between companies i and j. This definition allows looking at the structure of the network as a weighted graph where the links have certain weights associated with them, representing less or more important relationships. Transactions between any two parties add to the associated link weights in terms of volume. The elements w_{ij} of the weighted connectivity matrix B denote the overall number of transactions between companies i and j.

Additionally, our data set allows us to construct directed graphs where the links follow the flow of the money, such that a link is incoming to the receiver i and outgoing from the payer j. The matrix C is a weighted-directed graph where the links follow the flow of the money, such that a link is incoming to the receiver and outgoing from the sender of the payment. For this case we have two more matrices: in-degree and out-degree. The choice of usage of the matrix representation depends on the focus of the analysis.

The data set contains 3.4 million electronic company-to-company domestic payments of the full calendar year 2014, including data of 16 613 companies. This network shares typical structural characteristics known in other complex networks: degree distributions follow a power law, low clustering coefficient, and low average shortest path length. The average degree of the Estonian network of payments is $\langle k \rangle = 20$, the maximal degree is 345, and the diameter is 29. The average betweenness of the links is 40 while it is 110 for the nodes. The average shortest path length $\langle l \rangle = 7.1$. The network is a small world with 7.1 degrees of separation, which means that on average any company can be reached by another only in a few steps. Our network has low connectivity but is densely connected. The network also displays scale-free properties in its degree distributions. This scale-free structure indicates that few companies in Estonia trade with many parties while the majority trade with only few.

The network has a low average clustering coefficient of 0.18 and shows disassortative mixing. The detailed information of our network's full topologic structure and its basic characteristics can be found in our previous paper [16]. In that study, we performed an analysis to reveal the robustness of our network derived from its scale-free structure. We found that the network is resilient to random removal of the nodes but is vulnerable to targeted removal of nodes. The percolation threshold is 6%, and this means that a small portion of economic entities maintains the whole network unified. We found that most of the influential companies in the network are not necessarily the most connected ones and that a considerable number of companies who have high transactional activities have weak influence on the economic network as a whole.

3. LITERATURE DISCUSSION

Real networks, which are organized in a complex topological structure, show a large heterogeneity in the capacity and intensity of the connections (the weight of the links) [17]. Recently, many features of weighted networks have been studied, for example, the relationship between the node degree and node strength [18], degree correlations and perturbations [19], node correlations [20], and dynamical properties of nodes and degrees [21]. The study of highly interconnected systems became an important area of multidisciplinary research in network science involving physics, mathematics, biology, and social sciences, but recently the interest has shifted towards weighted networks. In the last 20 years, a large set of measures and metrics which combine topological and weighted observables has been proposed to characterize the statistical properties of nodes and links and to investigate the relationships between the weighted quantities and underlying network structures. Barrat et al. [17] presented a quantitative and general approach to understanding the complex architecture of weighted networks. They studied representative examples of social and large infrastructure systems, and defined specific metrics considering the weights of nodes and links in order to investigate the correlations among the weighted quantities and the underlying topological structure of these networks. They showed that a more complete view of complex networks is provided by the study of the interactions defining the links of these systems.

Zemp et al. [22] developed new versions of some measures for directed and/or weighted networks in order to take the importance of nodes into account. They showed that the use of their measures avoids systematic biases created by a higher node density and larger weights of the links. Newman [23] showed that weighted networks could be analysed by using a simple mapping from a weighted network to an unweighted multigraph, which allows using standard techniques for unweighted and weighted networks.

Network-based approaches are very useful and provide means for monitoring complex economic systems. They may also help in ensuring better control in managing and governing these systems. Regarding applications of economic networks and other recent studies, Souma et al. [24] studied a shareholder network of Japanese companies. In this study, the authors analysed the growth of companies through the analysis of the network's dynamics. A similar work by Rotundo and D'Arcangelis [25] dealt with the relationships of shareholders in the Italian stock market. Reyes et al. [26] made a weighted network analysis focused on using random walk betweenness centrality to study why high-performing Asian economies had higher economic growth than Latin American economies in 1980–2005. Other relevant studies on economic networks concentrate on the regional investment and ownership networks [27,28]. In these networks, European company-to-company foreign direct investment stocks show a power-law distribution with the number of employees in the investing company and in the company invested in, and with the volume of in- and outgoing investments of both companies. This power-law feature allows predicting the investments that will be received or made in specific regions, based on the connectivity and transactional activity of the companies. Nakano and White [29] showed that analytic concepts and methods related to complex networks can help to uncover structural factors that may influence the price formation for empirical market-link formations of economic agents.

Another interesting line of research is related to network topology as a basis for investigating money flows of customer-driven banking transactions. A few papers describe the actual topologies observed in different financial systems [30–32]. Other similar studies focus on economic shocks, robustness, and growth in economic or social complex networks [33–35]. Interesting reviews of complex network models and methods present the applications to socioeconomic issues [36,37].

4. METHODS

The degree of a node is defined as

$$k_i = \sum_{j\in\zeta(i)} a_{ij}, \tag{1}$$

where the sum goes over the set $\zeta(i)$ of neighbours of i. For example, $\zeta(i)=\{j\,|\,a_{ij}=1\}$. The degree of a node (company) refers to the number of payments linked to it.

Two relevant characteristics of a node occur in a directed network: the number of links that end at a node and the number of links that start from a node. These quantities are known as the in-degree k^d and out-degree k^o of a node, and we define them as

$$k^d = \sum_{j\in\zeta(i)} a_{ij}^d\ ,\ k^o = \sum_{j\in\zeta(i)} a_{ij}^o. \tag{2}$$

Weights w_{ij} of the links i and j in a network show the importance of each link. The strength s_i of the nodes is the sum of the weights of all the links. In our network, the strength measures the overall transaction value/volume for any given node, and is defined by the formula

$$s_i = \sum_{j\in\zeta(i)} w_{ij}, \tag{3}$$

where the sum runs over the set $\zeta(i)$ of neighbours of i.

For a given node i with connectivity k_i and strength s_i, the weights of the links might be of the same order of magnitude s_i/k_i, or they can be distributed heterogeneously with some links predominating over others. Then, the participation ratio is defined as follows:

$$H_2^w(i) = \sum_{j \in \zeta(i)} \left[\frac{w_{i,j}}{s_i^w} \right]^2, \tag{4}$$

or, equivalently,

$$H_2^c(i) = \sum_{j \in \zeta(i)} \left[\frac{c_{i,j}}{s_i^c} \right]^2. \tag{5}$$

We define the participation rates to separate outgoing and incoming links. Then, the average participation ratio is calculated as

$$H_2^w = \frac{1}{N} \sum_i H_2^w(i), \tag{6}$$

and

$$H_2^c = \frac{1}{N} \sum_i H_2^c(i), \tag{7}$$

respectively.

We calculate the participation ratio as a function of a company's inverse degree, where the objective is to identify the links that are used more often. If a low number of weights are dominant, then H_2^w is close to 1, but if all the weights are of the same order of magnitude, then $H_2^w \sim 1/k_i$. The value of H_2^w close to 1 indicates the existence of preferential interactions between the nodes, meaning that companies prefer to transact with certain companies.

5. RESULTS

5.1. Patterns of payments

The general characteristics and statistics of the Estonian network of payments are listed in Tables 1 and 2.

In this section, we will focus on the structure of our network and its time evolution. The objective of this effort is to identify structural changes and compare the emerging patterns. Figure 1a shows the monthly volume of transactions during 2014 while Fig. 1b displays the total number of transactions. Figure 1c shows the monthly average number of active links as a function of time. Figure 1b,c show that the number of transactions decreases dramatically in the third quarter of the year, while the number of active links decreases already in the second quarter. Also, these plots show that the number of transactions and the active links increase in the last quarter of the year, suggesting that liquidity in the Estonian network of payments increases by the end of the year through increased transaction volumes and payments, and higher than the usual number of active counterparties. It is interesting that the concentration in the volume of payments is high from August till the end of the year, while the number of payments diminishes dramatically in the same period of time. These observations indicate that the average number of active companies has decreased 20%, while the volume of transactions has increased 14% and the number of transactions has decreased 66% by the end of the year (compared with the beginning of the year). This indicates that companies in Estonia manage higher volumes of money at the end of the year than at the beginning of the year, while not all the companies remain active by the end of the year. A full explanation of this pattern of financial liquidity is not possible due to the lack of complete information about the overall financial and commercial activities of the companies in this network. Nonetheless, there are some possible explanations for these patterns. For example, these patterns could be highly affected by business cycles of payments, or by seasonal effects on the liquidity of companies, or by macroeconomic variations such as changes in the monetary policy of the Euro area and Estonia. Another explanation for the increased volume of transactions and

Table 1. Network characteristics

Companies analysed	16 613
Total number of payments analysed	2 617 478
Value of transactions	3 803 462 026*
Average value of transaction per customer	87 600*
Maximum value of transaction	121 533*
Minimum value of transaction (aggregated in the whole year)	1000*
Average volume of transaction per company	60
Maximum volume of transaction per company	24 859
Minimum volume of transaction per company (aggregated in the whole year)	20

* All the money amounts are expressed in monetary units and not in currencies in order to protect the confidentiality of the data set. The purpose of showing monetary units is to provide a notion of the proportion of quantities and not to show exact amounts of money.

Table 2. Summary of statistics

Statistic	Value	Components	Number of nodes
Undirected links	43 375	GCC	15 434
$\langle k \rangle$	20	DC	1179
γ^o	2.39	GSCC	3987
γ^i	2.49	GOUT	6054
γ	2.45	GIN	6172
$\langle C \rangle$	0.183	Tendrils	400
$\langle l \rangle$	7.1	Cutpoints	1401
T	0.13	Bi-component	4404
D	29	*k*-core	1081
$\langle \sigma \rangle$ (nodes)	110		
$\langle \sigma \rangle$ (links)	40		

$\langle k \rangle$ = average degree, γ^o = scaling exponent of the out-degree empirical distribution, γ^i = scaling exponent of the in-degree empirical distribution, γ = scaling exponent of the connectivity degree distribution, $\langle C \rangle$ = average clustering coefficient, $\langle l \rangle$ = average shortest path length, T = connectivity %, D = diameter, $\langle \sigma \rangle$ = average betweenness, GCC = Giant Connected Component, DC = Disconnected Component, GSCC = Giant Strong Connected Component, GOUT = Giant Out Component, GIN = Giant In Component.

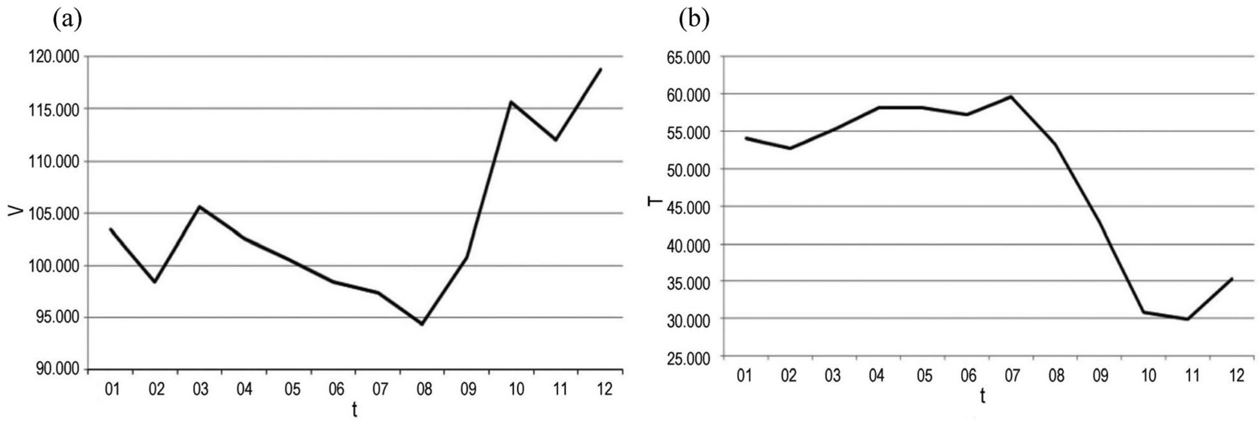
increased liquidity at the end of the year is that there might be a generalized release of delayed payments, like when companies try to spend the remaining balances of their annual budgets.

5.2. Strength and degree of nodes

We calculated the probability $P(s)$ that a company has k outgoing and incoming links. As per Fig. 2, the distribution of the out-degree volume (strength) follows a power-law decay

$$P(s) \sim s^{-2.32}, \tag{8}$$

where the scaling exponent is 2.32. There are some deviations from the power-law behaviour but they are sufficiently small. A similar distribution was also found in the in-degree volume (strength) distribution [16]. The power-law tail signals that the probability of finding companies paying out very large quantities of money is small. Moreover, while the companies have an absolute freedom in choosing how much money to pay or the counterparties who they interact with, the overall system obeys a scaling law, which is a particular property observed in critical phenomena and in highly interactive self-organized systems.



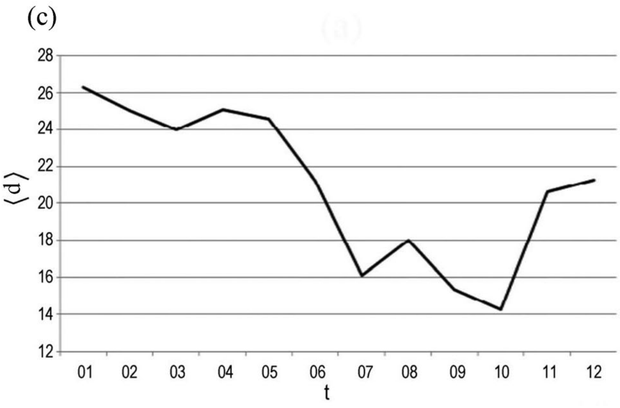


Fig. 1. Time evolution activity of patterns (payments and volumes). (a) Monthly trading volumes of payments V. (b) Monthly number of transactions T. (c) Average degree $\langle d \rangle$ versus time t. The x-axis represents the number of the month of the year 2014.

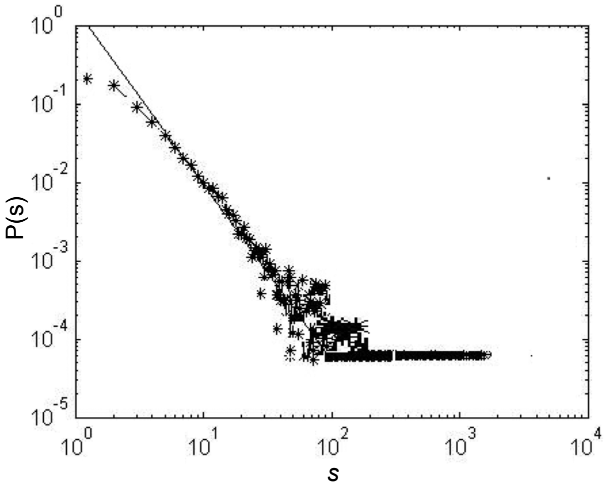


Fig. 2. Volume out-strength distribution.

We also analyse the bond between the strength and degree of a node. Figure 3a,b depict the volume and value (in and out) strengths as functions of degree of both outgoing and incoming links (in-degree and out-degree). The strength s is normalized by dividing it over the average link weight $\langle w_{ij} \rangle$. The following power-law relationship exists between the strength and the degree:

$$s(k) \sim k^{\alpha}, \tag{9}$$

where α is the coefficient of this scaling distribution. The power-law fit of Fig. 3a has an exponent $\alpha_{\text{vol}} = 1.5$, when volume is used as the weight, and $\alpha_{\text{val}} = 2.4$ when the value is used instead. These values imply that the out-strength of nodes s_{no} and in-strength of nodes s_{ni} grow faster than the degree k of a node, as seen in Fig 3a. It means that the most connected companies execute a higher number of payments with higher values of money than would be suggested only by their degree. This indicates that if a company has twice as many payments (out links) as another company, it could be expected that this company sends three times the number of payments, and almost five times the total value of payments. Figure 3b indicates that the relationships between in-degree and in-strength show similar trends to the out-degree and out-strength cases seen in Fig. 3a.

The strength of a node scales with its degree k, indicating that highly connected companies have payments of higher weight. The strength of a company grows generally faster than its degree. In other words, highly connected companies not only have many payments, but their payments also have a higher than average weight. This observation agrees with the fact that big companies are better equipped to handle large amounts of payments with higher amounts of money. Comparable results were found in the cargo ship movements network [38] and in the airport network [17], which may hint at a generic pattern in such large-scale networks.

5.3. Participation ratio

Figure 4b shows a plot of the participation ratio H_2^c as a function of the inverse degree of the nodes. The plot shows the links that are used more often than others. For example, for a degree up to 10 $H_2^c(i) \sim 1/k_i$ and for higher degrees the participation ratio is higher than the inverse degree, suggesting there is a disposition in the direction of preferential trading with specific counterparties. Figure 4b shows the average participation ratio during the whole year for outgoing payments and in-

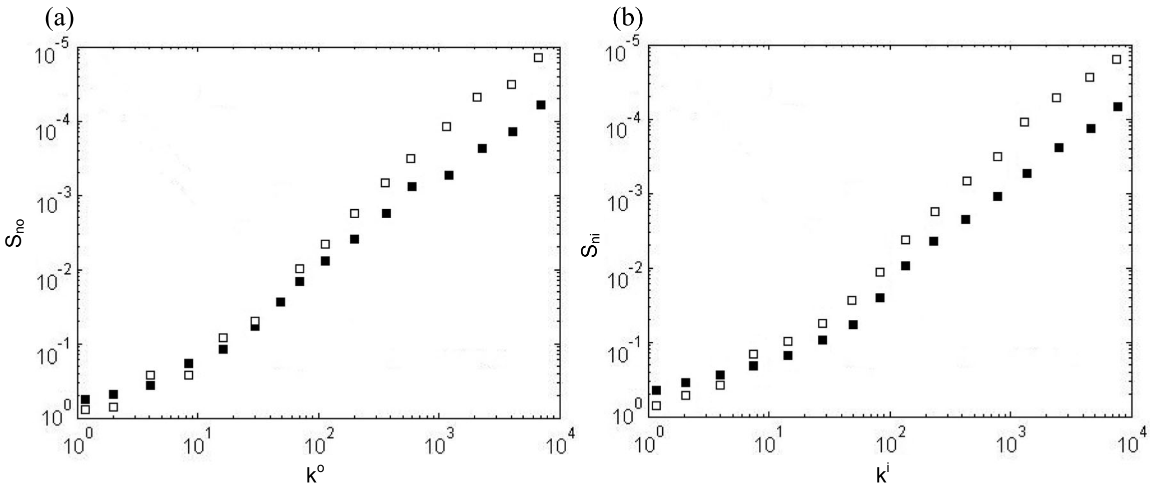


Fig. 3. Distributions of strength. (a) Node out-strength as a function of degree. (b) Node in-strength as a function of degree. Empty squares represent the value of payments and full squares represent the number of payments.

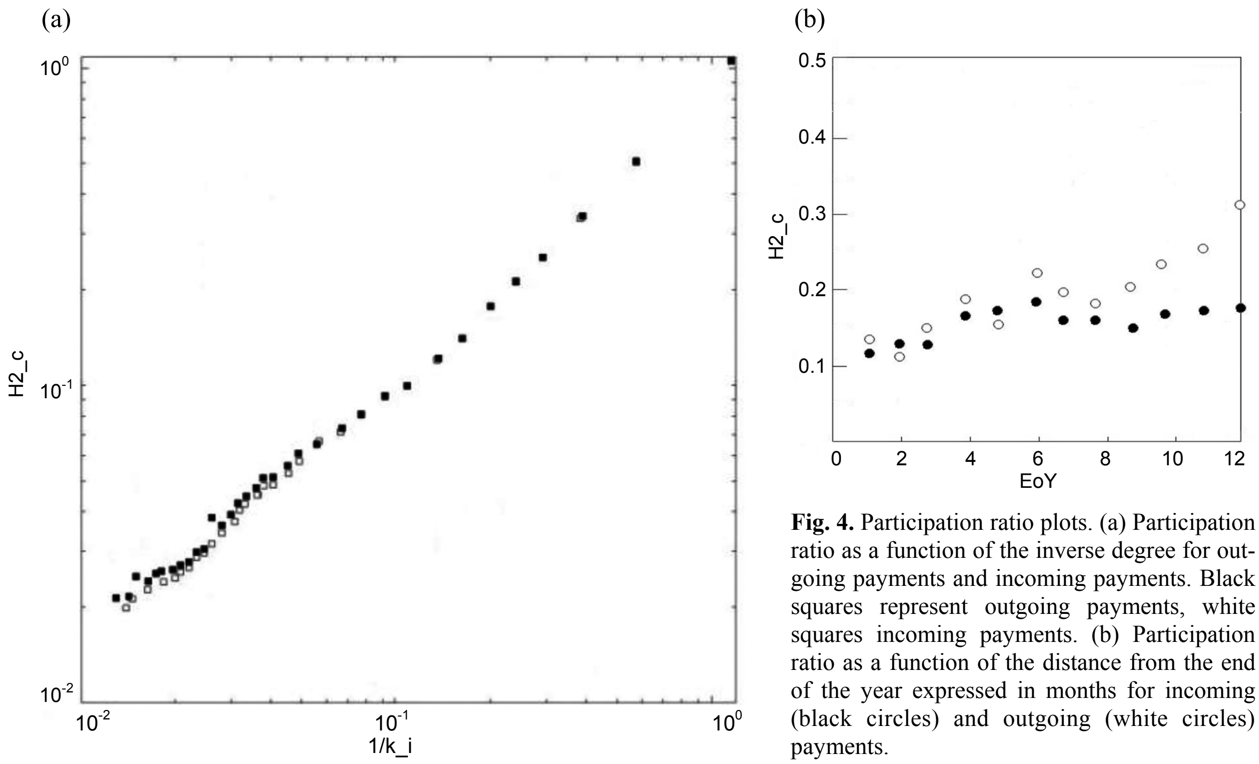


Fig. 4. Participation ratio plots. (a) Participation ratio as a function of the inverse degree for outgoing payments and incoming payments. Black squares represent outgoing payments, white squares incoming payments. (b) Participation ratio as a function of the distance from the end of the year expressed in months for incoming (black circles) and outgoing (white circles) payments.

coming payments. By the end of the year the participation ratio for all the payments decreases. Particularly, the participation ratio of the outgoing payments decreases dramatically. This reveals that the preferential linking is limited. By the end of the year, the preference for trading with only certain counterparties becomes less important. This could be caused by an increased payments/liquidity tendency that could potentially be driven by generalized unspent company annual budgets or delayed payments that were completed before the year ended.

6. CONCLUSIONS

In this study, we explored the relations between weighted quantities and their network underlying structures. We investigated the strength of interactions (number of payments and the volumes of payments) and the interconnectivities among these interactions in the Estonian network of payments by the realization of particular experiments, calculating specific metrics, and revealed interesting microstructural features.

We detected a clear pattern of structural changes over the analysed period in the network degree and number of payments decreasing by the end of the year, while the volume of payments increased. This indicates that Estonian companies handle higher volumes of cash flows at the end of the year than at the beginning of the year, while not all the companies remained active by the end of the year.

Scale-free properties were determined between the strengths and the degree values. We found that the most connected companies executed a higher number of payments with higher values of money than what would be suggested only by their degree (the out-strength of nodes and in-strength of nodes grow faster than the degree of a node).

It is important to continue observing, describing, and studying the structures and characteristics of economic complex networks in order to be able to understand their underlying processes and to detect patterns that could be useful for predicting or forecasting events and trends. The addition of evidence through empirical studies in favour of economic networks represents an important step towards the knowledge on the universality and the understanding of the complexity of economic systems.

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Eesti maksete võrgustiku sidemete ja sõlmpunktide statistilised eriomadused

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Selles töös uurisime elementidevaheliste seoste tugevust (maksete seoste kaalu ja maksete mahtu) Eesti maksete võrgustikus, viies selleks läbi spetsiaalseid eksperimente. Uurisime võrgustiku konkreetseid statistilisi näitajaid, mis ühendavad seoste tugevuse suhete topoloogia sõlmede ja nende erikaaluga, eesmärgiga minna sügavamale võrgustiku topoloogilisest arhitektuurist ning välja tuua selle kompleksstruktuuri aspekte. Lisaks leidsime skaalata omadusi tugevuse ja järkude väärtuste vahel. Samuti täheldasime analüüsitud perioodi jooksul strukturaalsete muutuste selget mustrit võrgustikus.