Proceedings of the Estonian Academy of Sciences,
2019, **68**, 1, 43–54
https://doi.org/10.3176/proc.2019.1.06
Available online at www.eap.ee/proceedings

MATHEMATICS

On generalized fuzzy sets in ordered LA-semihypergroups

Muhammad Gulistan[a], Naveed Yaqoob[b], Seifedine Kadry[c*], and Muhammad Azhar[a]

[a] Department of Mathematics and Statistics, Hazara University, Mansehra, 21130, Pakistan
[b] Department of Mathematics, College of Science Al-Zulfi, Majmaah University, Al-Zulfi 11932, Saudi Arabia
[c] Department of Mathematics and Computer Science, Faculty of Science, Beirut Arab University, P.O. Box 11-5020 Beirut, Lebanon

Received 3 August 2018, revised 5 November 2018, accepted 22 November 2018, available online 10 January 2019

Abstract. Using the notion of generalized fuzzy sets, we introduce the notions of generalized fuzzy hyperideals, generalized fuzzy bi-hyperideals, and generalized fuzzy normal bi-hyperideals in an ordered non-associative and non-commutative algebraic structure, namely an ordered LA-semihypergroup, and we characterize these hyperideals. We provide some results related to the images and preimages of generalized fuzzy hyperideals in ordered LA-semihypergroups.

Key words: ordered LA-semihypergroups, generalized fuzzy sets, generalized fuzzy hyperideals.

1. INTRODUCTION

In 1934, Marty [1] gave the concept of hypergroups. The difference between a classical algebraic structure and hyperstructures is that the composition of two elements is an element, while in an algebraic hyperstructures, the composition of two elements is a set. A recent book on hyperstructures [2] pointed out their applications in rough set theory, cryptography, codes, automata, probability, geometry, lattices, binary relations, graphs, and hypergraphs. Another book [3] is devoted especially to the study of hyperring theory. Bonansinga and Corsini [4], Corsini and Cristea [5], Davvaz [6] and Hasankhani [7] added many results to hyperstructure theory. Recently, Hila and Dine [8] introduced the notion of LA-semihypergroups which is the generalization of semigroups, semihypergroups, and LA-semigroups. Further, Yaqoob et al. [9] studied the concept of intra-regular LA-semihypergroups with pure left identity and Yousafzai and Corsini [10] considered some characterization problems in LA-semihypergroups. The basic idea of ordered semihypergroups was introduced by Heidari and Davvaz in [11], where they used a binary relation $\leq$ in semihypergroup $(H,\circ)$ such that the binary relation is a partial order and the structure $(H,\circ,\leq)$ is known as ordered semihypergroups. The ordering in LA-semihypergroup was introduced by Yaqoob and Gulistan in [12].

The most appropriate theory for dealing with uncertainties is the theory of fuzzy sets developed by Zadeh [13]. Murali [14] defined the concept of belongingness of a fuzzy point to a fuzzy subset under a natural equivalence on a fuzzy subset. The idea of quasi-coincidence of a fuzzy point with a fuzzy set defined in

* Corresponding author, skadry@gmail.com

[15] played a vital role in the generation of some different types of fuzzy subgroups. It is worth mentioning that Bhakat and Das [16] gave the concept of (α,β)-fuzzy subgroups by using the "belongs to" relation $\in$ and "quasi coincident with" relation q between a fuzzy point and a fuzzy subgroup, and introduced the concept of an $(\in,\in\vee q)$-fuzzy subgroup, where $\alpha,\beta\in\{\in,q,\in\vee q,\in\wedge q\}$ and $\alpha\neq\in\wedge q$. In particular, a $(\in,\in\vee q)$-fuzzy subgroup is an important and useful generalization of Rosenfeld's fuzzy subgroup.

Fuzzy hyperideals of ordered semihypergroups were investigated by Pibaljommee et al. [17]. Further, ordered semihypergroups in terms of fuzzy hyperideals were considered by Tang et al. [18]. Recently, Azhar et al. [19] discussed fuzzy hyperideals of ordered LA-semihypergroups. More recently, Azhar et al. [20] gave the concept of $(\in,\in\vee q_k)$-fuzzy hyperideal of an ordered LA-semihypergroup by using the ordered fuzzy points and investigated related properties. Shabir and Mahmood [21] characterized semihypergroups by the properties of $(\in_\gamma,\in_\gamma\vee q_\delta)$-fuzzy hyperideals, see also [22,23].

In this paper, we characterize ordered LA-semihypergroups by the properties of their $(\in_\gamma,\in_\gamma\vee q_\delta)$-fuzzy hyperideals, $(\in_\gamma,\in_\gamma\vee q_\delta)$-fuzzy bi-hyperideals, and $(\in_\gamma,\in_\gamma\vee q_\delta)$-fuzzy normal bi-hyperideals. We show that the set of all $(\in_\gamma,\in_\gamma\vee q_\delta)$-fuzzy hyperideals becomes an ordered LA-semihypergroup. We present results on images and preimages of $(\in_\gamma,\in_\gamma\vee q_\delta)$-fuzzy hyperideals of ordered LA-semihypergroups.

2. PRELIMINARIES AND BASIC DEFINITIONS

In this section, we recall certain definitions and results needed for our study.

Definition 1. *A map $\circ: H\times H\to\mathscr{P}^*(H)$ is called a hyperoperation or a join operation on the set H, where H is a non-empty set and $\mathscr{P}^*(H)=\mathscr{P}(H)\backslash\{\emptyset\}$ denotes the set of all non-empty subsets of H. A hypergroupoid is a set H together with a (binary) hyperoperation.*

If A and B are two non-empty subsets of H, we denote

$$A\circ B=\bigcup_{a\in A,b\in B} a\circ b,\quad a\circ A=\{a\}\circ A,\ \text{ and } a\circ B=\{a\}\circ B.$$

Definition 2. [8] *A hypergroupoid $(H,\circ)$ is called an LA-semihypergroup if for all $x,y,z\in H$, $(x\circ y)\circ z=(z\circ y)\circ x$.*

The law $(x\circ y)\circ z=(z\circ y)\circ x$ is called a left invertive law. Every LA-semihypergroup satisfies the law $(x\circ y)\circ(z\circ w)=(x\circ z)\circ(y\circ w)$ for all $x,y,z,w\in H$. This law is known as medial law (cf. [8]).

Definition 3. [9] *Let H be an LA-semihypergroup. An element $e\in H$ is called*
(i) left identity (resp., pure left identity) if for all $a\in H$, $a\in e\circ a$ (resp., $a=e\circ a$),
(ii) right identity (resp., pure right identity) if for all $a\in H$, $a\in a\circ e$ (resp., $a=a\circ e$),
(iii) identity (resp., pure identity) if for all $a\in H$, $a\in e\circ a\cap a\circ e$ (resp., $a=e\circ a\cap a\circ e$).

Definition 4. [12] *Let H be a non-empty set and $\leq$ be an ordered relation on H. The triplet $(H,\circ,\leq)$ is called an ordered LA-semihypergroup if the following conditions are satisfied:*
(1) $(H,\circ)$ is an LA-semihypergroup,
(2) $(H,\leq)$ is a partially ordered set,
(3) for every $a,b,c\in H$, $a\leq b$ implies $a\circ c\leq b\circ c$ and $c\circ a\leq c\circ b$, where $a\circ c\leq b\circ c$ means that for $x\in a\circ c$ there exist $y\in b\circ c$ such that $x\leq y$.

Definition 5. [12] *If $(H,\circ,\leq)$ is an ordered LA-semihypergroup and $A\subseteq H$, then $(A]$ is the subset of H defined as $(A]=\{t\in H: t\leq a,$ for some $a\in A\}$.*

Definition 6. [12] *A non-empty subset A of an ordered LA-semihypergroup $(H,\circ,\leq)$ is called an LA-subsemihypergroup of H if $(A\circ A]\subseteq(A]$.*

Definition 7. [12] *A non-empty subset A of an ordered LA-semihypergroup $(H,\circ,\leq)$ is called a right (resp., left) hyperideal of H if*

(1) $A\circ H\subseteq A$ (resp., $H\circ A\subseteq A$),

(2) for every $a\in H$, $b\in A$ and $a\leq b$ implies $a\in A$.

If A is both right hyperideal and left hyperideal of H, then A is called a hyperideal (or two-sided hyperideal) of H.

Definition 8. [19] *Let $x\in H$, then $A_x=\{(y,z)\in H\circ H : x\leq y\circ z\}$. Let f and g be two fuzzy subsets of an ordered LA-semihypergroup H, then $f*g$ is defined as*

$$(f*g)(x)=\begin{cases}\bigvee\limits_{(y,z)\in A_x}\{f(y)\wedge g(z)\} & \text{if } x\leq y\circ z\text{, for some } y,z\in H\\ 0 & \text{otherwise.}\end{cases}$$

Let $\mathscr{F}(H)$ denote the set of all fuzzy subsets of an ordered LA-semihypergroup.

Definition 9. [19] *Let $(H,\circ,\leq)$ be an ordered LA-semihypergroup. A fuzzy subset $f:H\to[0,1]$ is called a fuzzy LA-subsemihypergroup of H if the following assertions are satisfied:*

(i) $\bigwedge\limits_{z\leq a\circ b} f(z)\geq\min\{f(a),f(b)\}$,

(ii) if $a\leq b$ implies $f(a)\geq f(b)$, for every $a,b\in H$.

Definition 10. [19] *Let $(H,\circ,\leq)$ be an ordered LA-semihypergroup. A fuzzy subset $f:H\to[0,1]$ is called a fuzzy right (resp., left) hyperideal of H if*

(1) $\bigwedge\limits_{z\leq a\circ b} f(z)\geq f(a)$ (resp., $\bigwedge\limits_{z\leq a\circ b} f(z)\geq f(b)$),

(2) $a\leq b$ implies $f(a)\geq f(b)$, for every $a,b\in H$.

If f is both a fuzzy right hyperideal and a fuzzy left hyperideal of H, then f is called a fuzzy hyperideal of H.

Proposition 1. [19] *Let H be an ordered LA-semihypergroup. Then the set $(\mathscr{F}(H),\circ)$ becomes an ordered LA-semihypergroup, where $\mathscr{F}(H)$ denotes the family of all fuzzy subsets in H.*

Definition 11. [21] *For a fuzzy point a_t and a fuzzy subset f of H, we say that*

(i) $a_t\in_\gamma f$ if $f(a)\geq t>\gamma$,

(ii) $a_t q_\delta f$ if $f(a)+t>2\delta$,

(iii) $a_t\in_\gamma\vee q_\delta f$ if $a_t\in_\gamma f$ or $a_t q_\delta f$.

Definition 12. [21] *Let $\gamma,\delta\in[0,1]$ be such that $\gamma<\delta$. For any subsets A and B of H such that $B\subseteq A$, we define $\chi^\delta_{\gamma B}$ be the fuzzy subset of H by $\chi^\delta_{\gamma B}(x)\geq\delta$ for all $x\in B$ and $\chi^\delta_{\gamma B}(x)\leq\gamma$ if $x\notin B$. Clearly, $\chi^\delta_{\gamma B}$ is the characteristic function of B if $\gamma=0$ and $\delta=1$.*

Definition 13. [21] *For any fuzzy subsets f,g of $\mathscr{F}(H)$, by $f\subseteq\vee q_{(\gamma,\delta)}g$, we mean that $x_r\in_\gamma f$ implies $x_r\in_\gamma\vee q_\delta g$ for all $x\in H$ and $r\in(\gamma,1]$.*

Definition 14. [21] *$f=_{(\gamma,\delta)}g$ if $f\subseteq\vee q_{(\gamma,\delta)}g$ and $g\subseteq\vee q_{(\gamma,\delta)}f$.*

Lemma 1. [21] *Let f and g be two fuzzy subsets. Then $f\subseteq\vee_{q(\gamma,\delta)}g$ if and only if $\max\{g(a),\gamma\}\geq\min\{f(a),\delta\}$, where $\delta,\gamma\in(0,1]$ such that $\gamma<\delta$.*

Corollary 1. [21] *Let f, g, and h be fuzzy subsets such that $f\subseteq\vee_{q(\gamma,\delta)}g$ and $g\subseteq\vee_{q(\gamma,\delta)}h$ implies $f\subseteq\vee_{q(\gamma,\delta)}h$.*

3. $(\in_\gamma, \in_\gamma \vee q_\delta)$-FUZZY SETS IN ORDERED LA-SEMIHYPERGROUPS

In this section, we discuss some basic properties of $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy sets in ordered LA-semihypergroups.

Definition 15. *A fuzzy subset f of H is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy LA-subsemihypergroup of H*

(C_1) if $x_{t_1} \in_\gamma f$ and $y_{t_2} \in_\gamma f \Longrightarrow (z)_{\min\{t_1,t_2\}} \in_\gamma \vee q_\delta f$ for every $z \in x \circ y$ such that $\gamma < \delta$;

(C_2) if $y \leq x$ and $x_t \in_\gamma f \Longrightarrow y_t \in_\gamma \vee q_\delta f$ for all $x, y \in H$, $t, t_1, t_2 \in (\gamma, 1]$ such that $\gamma < \delta$.

Example 1. Let $H = \{x, y, z\}$ be an LA-semihypergroup defined as

$\circ$	x	y	z
x	$\{x,y\}$	$\{x,y\}$	z
y	$\{x,z\}$	$\{x,z\}$	z
z	z	z	z

and the order relation defined as $\leq$: $\{(x,x),(y,y),(z,z),(z,x),(z,y)\}$. Then $(H, \circ, \leq)$ is an ordered LA-semihypergroup. If we define $f(x) = 0.8$, $f(y) = 0.7$, $f(z) = 0.6$ and $t_1 = 0.3$, $t_2 = 0.4$, $t = 0.5$, $\gamma = 0.2$, $\delta = 0.3$, then clearly f is an $(\in_{0.2}, \in_{0.2} \vee q_{0.1})$-fuzzy LA-subsemihypergroup of H.

Theorem 1. *Let f be a fuzzy subset in H. Then f is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy LA-subsemihypergroup of H if and only if the following conditions hold:*

(1) $\max\{\inf_{z \in x \circ y} f(z), \gamma\} \geq \min\{f(x), f(y), \delta\}$,

(2) if $y \leq x$, then $\max\{f(y), \gamma\} \geq \min\{f(x), \delta\}$, where δ and $\gamma \in (0,1]$ such that $\gamma < \delta$.

Proof. Let f be a fuzzy subset in H such that it is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy LA-subsemihypergroup. Assume that there exist $x, y \in H$ such that (1) $\max\{\inf_{z \in x \circ y} f(z), \gamma\} < \min\{f(x), f(y), \delta\}$. Then there exist $z \in x \circ y$ such that

$$\max\{f(z), \gamma\} < \min\{f(x), f(y), \delta\}.$$

Choose $t \in (0,1]$ such that $\max\{\inf_{z \in x \circ y} f(z), \gamma\} < t \leq \min\{f(x), f(y), \delta\}$. Then $\max\{\inf_{z \in x \circ y} f(z), \gamma\} < t \Rightarrow \inf_{z \in x \circ y} f(z) < t < \gamma$. It follows that $(z)_t \overline{\in_\gamma \vee q_\delta} f$ for $z \in x \circ y$. On the other hand, if $(t \leq \min\{f(x), f(y), \delta\})$, we get $(f(x) \geq t > \gamma, \quad f(y) \geq t > \gamma)$, which implies $x_t \in_\gamma f$ and $y_t \in_\gamma f$ but $z_t \overline{\in_\gamma \vee q_\delta} f$ for $z \in x \circ y$, which is a contradiction to the hypothesis. Hence (1) is valid.

Again, assume that from (2) if $y \leq x$, then $\max\{f(y), \gamma\} < t \leq \min\{f(x), \delta\}$ for any $t \in (0,1]$. We have $f(x) \geq t > \gamma$ so $x_t \in_\gamma f$ but $y_t \overline{\in_\gamma \vee q_\delta} f$, which is a contradiction to the hypothesis. Hence (2) is valid.

Conversely, assume that (1) is valid and there exist $x \in H$ and $t_1, t_2 \in (0,1]$ such that $x_{t_1} \in_\gamma f$, and $y_{t_2} \in_\gamma f$. This implies that $f(x) \geq t_1 > \gamma, f(y) \geq t_2 > \gamma$.

So from (1) $\max\{\inf_{z \in x \circ y} f(z), \gamma\} \geq \min\{f(x), f(y), \delta\} \geq \min\{t_1, t_2, \delta\}$. We have the following two cases:

(i) If $\min\{t_1, t_2\} \leq \delta$, then $\inf_{z \in x \circ y} f(z) \geq \min\{t_1, t_2\} > \gamma$. This implies that $z_{\min\{t_1,t_2\}} \in_\gamma f$.

(ii) If $\min\{t_1, t_2\} > \delta$, then $\inf_{z \in x \circ y} f(z) + \min\{t_1, t_2\} > 2\delta$. This implies that $z_{\min\{t_1,t_2\}} q_\delta f$.

Hence from the above cases we get $z_{\min\{t_1,t_2\}} \in_\gamma \vee q_\delta f$. Assume (2) is valid and $x \in H$ and $t \in (0,1]$ such that $x_{t_1} \in_\gamma f$. This implies that $f(x) \geq t > \gamma$. So from (2) if $y \leq x$, then $\max\{f(y), \gamma\} \geq \min\{f(x), \delta\} \geq \min\{t, \delta\}$. We have the following two cases:

(i) If $t \leq \delta$, then $f(y) \geq t_2 > \gamma$. This implies that $y_t \in_\gamma f$.

(ii) If $t > \delta$, then $f(y) + t > 2\delta$. This implies that $y_t q_\delta f$.

Hence from the above cases we get $y_t \in_\gamma \vee q_\delta f$. Thus f is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy LA-subsemihypergroup of H. □

Lemma 2. *Let $\emptyset \neq A \subseteq H$. Then A is an ordered LA-subsemihypergroup of H if and only if the characteristic function $\chi_{\gamma A}^{\delta}$ of A is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy LA-subsemihypergroup of H, where $\delta, \gamma \in (0,1]$ such that $\gamma < \delta$.*

Proof. The proof is straightforward. □

Definition 16. *The $\in_\gamma \vee q_\delta$-fuzzy level set for the fuzzy subset of f is defined as $[f]_t = \{x \in H : x_t \in_\gamma \vee q_\delta f\}$.*

Theorem 2. *A fuzzy subset f is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy LA-subsemihypergroup of H if and only if $\emptyset \neq [f]_t$ is an LA-subsemihypergroup of H.*

Proof. Let $\emptyset \neq [f]_t$ be an LA-subsemihypergroup of H. We have to show that f is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy LA-subsemihypergroup of H. Assume that $x, y \in H$ and $t \in (0,1]$, such that $(\max\{\inf_{z \in x \circ y} f(z), \gamma\} < t \leq \min\{f(x), f(y), \delta\})$. Then $(\max\{\inf_{z \in x \circ y} f(z), \gamma\} < t \Rightarrow \max\{f(z), \gamma\} < t$, so $f(z) < t < \gamma)$, i.e., $z_t \overline{\in_\gamma \vee q_\delta} f$. On the other hand, if $(t_1 \leq \min\{f(x), f(y), \delta\})$, then $(f(x) \geq t_1 > \gamma, f(y) \geq t_1 > \gamma)$, i.e., $x_{t_1,} \in_\gamma f$ and $y_{t_2} \in_\gamma f$, but $z_{t_1} \overline{\in_\gamma \vee q_\delta} f$, which is a contradiction to the hypothesis. Thus $(\max\{\inf_{z \in x \circ y} f(z), \gamma\} \geq \min\{f(x), f(y), \delta\})$. Also assume that $x, y \in H$ and $t \in (0,1]$, such that $(\max\{f(y), \gamma\} < t \leq \min\{f(x), \delta\})$. Then $(\max\{f(y), \gamma\} < t \Rightarrow \max\{f(y), \gamma\} < t$, so $f(y) < t < \gamma)$, i.e., $y_t \overline{\in_\gamma \vee q_\delta} f$. On the other hand, if $(t \leq \min\{f(x), \delta\})$, then $(f(x) \geq t > \gamma)$, i.e., $x_{t_1,} \in_\gamma f$, but $y_{t_1} \overline{\in_\gamma \vee q_\delta} f$, which is a contradiction to the hypothesis. Thus $(\max\{f(y), \gamma\} \geq \min\{f(x), \delta\})$. Hence f is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy LA-subsemihypergroup of H. Conversely, let f be an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy LA-subsemihypergroup of H. Then $x_t \in_\gamma [f]_t$ and $y_t \in_\gamma [f]_t$ imply that $((f(x) \geq t > \gamma, f(x) + t > 2\delta))$ and $((f(y) \geq t > \gamma, f(y) + t > 2\delta))$. Now by using the hypothesis we have $\max\{\inf_{z \in x \circ y} f(z), \gamma\} \geq \min\{f(x), f(y), \delta\} \geq \min\{t, t, \delta\} = t$. This shows that $z \in [f]_t$, for all $x \in [f]_t$ and $y \in [f]_t$. Also, $\max\{f(y), \gamma\} \geq \min\{f(x), \delta\} \geq \min\{t, \delta\} = t$ and we get $y \leq x$, then $y \in [f]_t$. Hence $[f]_t$ is an ordered LA-subsemihypergroup of H. □

Theorem 3. *The intersection of any two $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy LA-subsemihypergroups of H is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy LA-subsemihypergroup of H.*

Proof. Let f and g be two $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy LA-subsemihypergroups of H. We will show that $f \cap g =$ is also an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy LA-subsemihypergroup of H. We assume that $x_{t_1} \in_\gamma f \cap g$ and $y_{t_2} \in_\gamma f \cap g$. This implies that $x_{t_1} \in_\gamma f$, $x_{t_1} \in_\gamma g$ and $y_{t_2} \in_\gamma f$, $y_{t_2} \in_\gamma g$. Now using the fact that f and g are two $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy LA-subsemihypergroups of H, we have $z_{\min\{t_1, t_2\}} \in_\gamma \vee q_\delta f$ and $z_{\min\{t_1, t_2\}} \in_\gamma \vee q_\delta g$. This implies that $z_{\min\{t_1, t_2\}} \in_\gamma \vee q_\delta f \cap g$. Also if $y \leq x$, then assume that $x_{t_1} \in_\gamma f \cap g$. This implies that $x_{t_1} \in_\gamma f$, $x_{t_1} \in_\gamma g$. Now using the fact that f and g are two $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy LA-subsemihypergroups of H, we have $y_t \in_\gamma \vee q_\delta f$ and $y_t \in_\gamma \vee q_\delta g$. This implies that $y_t \in_\gamma \vee q_\delta f \cap g$. Hence $f \cap g$ is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy LA-subsemihypergroup of H. □

Definition 17. *A fuzzy subset f of H is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy left hyperideal of H if f satisfies (C_2) and $x_t \in_\gamma f, y \in H \Longrightarrow (z)_t \in_\gamma \vee q_\delta f$, $t \in (\gamma, 1]$ for all $z \in y \circ x$.*

Definition 18. *A fuzzy subset f of H is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy right hyperideal of H if f satisfies (C_2) and $x_t \in_\gamma f, y \in H \Longrightarrow (z)_t \in_\gamma \vee q_\delta f$, $t \in (\gamma, 1]$ for all $z \in x \circ y$.*

A fuzzy subset f is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy hyperideal if it is both an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy left and an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy right hyperideal of H.

Theorem 4. *For an ordered LA-semihypergroup H, the following conditions are equivalent:*
(i) f is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy left (resp., right) hyperideal of H,
(ii) $\mathscr{H} \circ f \subseteq \vee_{q(\gamma,\delta)} f$ (resp., $f \circ \mathscr{H} \subseteq \vee_{q(\gamma,\delta)} f$), where $\mathscr{H}(x) = 1$ and for all $x \in H$.

Proof. (i)⇒(ii) Let f be an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy left hyperideal of H and $a \in H$. Let us suppose that there exist $x, y \in H$ such that $a \in x \circ y$. In order to show that $\mathscr{H} \circ f \subseteq \vee_{q(\gamma,\delta)} f$, we have to show that
(1) $(\max\{f(a), \gamma\} \geq \min\{\mathscr{H} \circ f(a), \delta\})$,
(2) if $y \leq x$, then $(\max\{f(x), \gamma\} \geq \min\{\mathscr{H} \circ f(y), \delta\})$, where $\delta, \gamma \in (0,1]$ such that $\gamma < \delta$.
Let us consider (1)

$$(\mathscr{H} \circ f)(a) = \vee_{a \in x \circ y}[\min\{\mathscr{H}(x), f(y)\}] = \vee_{a \in x \circ y}[\min\{1, f(y)\}] = \vee_{a \in x \circ y} f(y).$$

Since f is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy left hyperideal of H, we have $\max\{\inf_{a\in x\circ y} f(a), \gamma\} \geq \min\{f(y), \delta\}$. In particular, $f(y) \leq f(a)$ for all $a \in x \circ y$. Hence $\vee_{a\in x\circ y} f(y) \leq f(a)$. Thus $\min\{\mathscr{H} \circ f(a), \delta\} = \min\{(1 \circ f)(a), \delta\} \leq \max\{f(a), \gamma\}$. If there do not exist $x, y \in H$ such that $a \in x \circ y$, then $\mathscr{H} \circ f(a) = (\mathscr{H} \circ f)(a) = 0 \leq f(a)$, so again we have $\min\{\mathscr{H} \circ f(a), \delta\} = \min\{(1 \circ f)(a), \delta\} \leq \max\{f(a), \gamma\}$.

Let us consider (2)

$$(\mathscr{H} \circ f)(a) = \vee_{a\in x\circ y}[\min\{\mathscr{H}(x), f(y)\}] = \vee_{a\in x\circ y}[\min\{1, f(y)\}] = \vee_{y\in a\circ b} f(y).$$

Since f is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy left hyperideal of H, we have $y \leq x$, then $(\max\{f(x), \gamma\} \geq \min\{f(y), \delta\})$. In particular, $f(y) \leq f(x)$. Hence $\vee_{y\in a\circ b} f(y) \leq f(x)$. Thus $\min\{f(y), \delta\} \leq \max\{f(x), \gamma\}$. If there do not exist $x, y \in H$ such that $a \in x \circ y$, then $\mathscr{H} \circ f(a) = (\mathscr{H} \circ f)(a) = 0 \leq f(a)$, so again we have $\min\{f(y), \delta\} \leq \max\{f(x), \gamma\}$. Thus $\mathscr{H} \circ f \subseteq \vee_{q(\gamma,\delta)} f$.

(ii)⇒(i) Let $x, y \in H$ and $a \in x \circ y$. Then $\inf\limits_{a\in x\circ y} f(a) \geq (\mathscr{H} \circ f)(a)$. We have

$$(\mathscr{H} \circ f)(a) = \bigvee_{a\in x\circ y}[\min\{\mathscr{H}(x), f(y)\}] \geq \min\{\mathscr{H}(x), f(y)\} = \min\{1, f(y)\} = f(y).$$

Consequently, $\inf_{a\in x\circ y} f(a) \geq f(y)$, which implies that $\max\{\inf_{a\in x\circ y} f(a), \gamma\} \geq \min\{f(y), \delta\}$. Also if $y \leq x$, then $f(x) \geq (\mathscr{H} \circ f)(y)$. We have

$$(\mathscr{H} \circ f)(a) = \bigvee_{a\in x\circ y}[\min\{\mathscr{H}(x), f(y)\}] \geq \min\{\mathscr{H}(x), f(y)\} = \min\{1, f(y)\} = f(y).$$

Consequently, $f(x) \geq f(y)$, which implies that $\max\{f(x), \gamma\} \geq \min\{f(y), \delta\}$. Hence f is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy left hyperideal of H. □

Theorem 5. *Let f be a fuzzy subset in H. Then f is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy left (resp., right) hyperideal of H if and only if*

$$\begin{aligned} \max\{\inf_{z\in x\circ y} f(z), \gamma\} &\geq \min\{f(y), \delta\}, \\ \textit{If } y &\leq x, \textit{ then } \max\{f(x), \gamma\} \geq \min\{f(y), \delta\}; \\ (\textit{resp., } \max\{\inf_{z\in x\circ y} f(z), \gamma\} &\geq \min\{f(x), \delta\}), \\ \textit{If } y &\leq x, \textit{ then } \max\{f(x), \gamma\} \geq \min\{f(y), \delta\}, \end{aligned}$$

where δ and $\gamma \in (0,1]$ such that $\gamma < \delta$.

Proof. Similar to the proof of Theorem 1. □

Theorem 6. *Let $\emptyset \neq A \subseteq H$. Then A is a left (resp., right) hyperideal of H if and only if the fuzzy characteristic function $\chi_\gamma^\delta A$ of A is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy left (resp., right) hyperideal of H, where $\delta, \gamma \in D(0,1]$ such that $\gamma < \delta$.*

Proof. The proof is straightforward. □

Theorem 7. *A fuzzy subset f of H is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy left (resp., right) hyperideal of H if and only if $\emptyset \neq [f]_t$ is a left (resp., right) hyperideal of H.*

Proof. Similar to the proof of Theorem 2. □

Theorem 8. *A fuzzy subset f is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy left (resp., right, two-sided) hyperideal of H if and only if the non-empty fuzzy level set $U(f;(t,\gamma)) = \{x \in H : f(x) \geq t > \gamma\}$ is a left (resp., right, two-sided) hyperideal of H, where $t, \gamma \in [0,1)$.*

Proof. The proof is straightforward. □

Lemma 3. *Let H be an ordered LA-semihypergroup and let f and g be an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy right hyperideal and $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy left hyperideal of H, respectively. Then $f \circ g \subseteq \vee_{q(\gamma,\delta)} f \cap g$.*

Proof. If $x \in H$ such that $x \notin y \circ z$, then $f(x) = 0$, so $(\max\{f(x), \gamma\} \geq \min\{f(x), \delta\})$, and if $y \leq x$, then $\max\{f(x), \gamma\} \geq \min\{f(y), \delta\})$, where $\delta, \gamma \in (0,1]$ such that $\gamma < \delta$. On the other hand, if $x \in y \circ z$ for some y and $z \in H$, then we have

$$\begin{aligned} \min\{f(x), \delta\} &= \min\{\vee_{x \in y \circ z}\{\min\{f(y), g(z)\}\}, \delta\} = \min\{\min\{f(y), g(z)\}, \delta\} \\ &= \min\{\min\{f(y), \delta\}, \min\{g(z), \delta\}\} \\ &\leq \min\{\max\{\inf_{x \in y \circ z} f(x), \gamma\}, \max\{\inf_{x \in y \circ z} f(x), \gamma\}\} \text{ by hypothesis} \\ &= \max\{f(x), \gamma\}, \end{aligned}$$

also

$$\begin{aligned} \min\{f(y), \delta\} &= \min\{\vee_{y \in p \circ z}\{\min\{f(p), g(z)\}\}, \delta\} = \min\{\min\{f(p), g(z)\}, \delta\} \\ &= \min\{\min\{f(p), \delta\}, \min\{g(z), \delta\}\} \\ &\leq \min\{\max\{\vee_{y \in p \circ z} f(y), \gamma\}, \max\{\vee_{y \in p \circ z} f(y), \gamma\}\} \text{ by hypothesis} \\ &= \max\{f(x), \gamma\}. \end{aligned}$$

Thus $f \circ g \subseteq \vee_{q(\gamma,\delta)} f \cap g$. □

Theorem 9. *Let H be an ordered LA-semihypergroup with pure left identity e. Then every $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy right hyperideal of H is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy left hyperideal of H.*

Proof. Let f be an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy right hyperideal of H, and let $x, y \in H$. Then $z \in x \circ y = (e \circ x) \circ y = (y \circ x) \circ e$,

$$\begin{aligned} \max\{\sup_{z \in x \circ y} f(z), \gamma\} &= \max\{\sup_{z \in (y \circ x) \circ e} f(z), \gamma\} \geq \min\{f(y), \delta\}, \\ \text{if } y &\leq x, \text{ then } \max\{f(y), \gamma\} \geq \min\{f(y), \delta\}. \end{aligned}$$

Hence f is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy left hyperideal of H and therefore f is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy hyperideal of H. □

Corollary 2. *Let H be an ordered LA-semihypergroup with pure left identity e. Then every $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy right hyperideal of H is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy hyperideal of H.*

Proof. The proof is straightforward. □

Theorem 10. *Let H be an ordered LA-semihypergroup and let $\{f_i\}_{i \in \wedge}$ be a family of $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy hyperideals of H. Then $\cap_{i \in \wedge} f_i$ is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy hyperideal of H.*

Proof. Similar to the proof of Theorem 3. □

Definition 19. *A fuzzy subset f of H is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy bi-hyperideal of H if for all $x, y, z \in H$ and $t_1, t_2 \in (0,1]$, it satisfies (C_1), (C_2) and $x_{t_1} \in_\gamma f$, $z_{t_2} \in_\gamma f$ implies that $w_{\min\{t_1,t_2\}} \in_\gamma \vee q_\delta f$ for all $w \in (x \circ y) \circ z$.*

Example 2. Let $H=\{e,x,y,z,w\}$ be an LA-semihypergroup as defined below:

$\circ$	e	x	y	z	w
e	e	x	y	z	w
x	y	z	z	$\{z,w\}$	w
y	x	z	z	$\{z,w\}$	w
z	z	$\{z,w\}$	$\{z,w\}$	$\{z,w\}$	w
w	w	w	w	w	w

and the order relation as $\leq$: $\{(e,e),(x,x),(y,y),(z,z),(w,e),(w,x),(w,y),(w,z),(w,w)\}$. Then $(H,\circ,\leq)$ is an ordered LA-semihypergroup. Define

$$f(a)=\begin{cases} 0.9 \text{ if } a=e \\ 0.8 \text{ if } a\in\{x,y\} \\ 0.6 \text{ if } a=z \\ 0.5\text{if } a=w \end{cases}$$

$t=t_1=0.3$, $t_2=0.4$, $\gamma=0.2$, and $\delta=0.3$. Then clearly f is an $(\in_{0.2},\in_{0.2}\vee q_{0.1})$-fuzzy bi-hyperideal of H.

Theorem 11. *For an ordered LA-semihypergroup H the following holds:*
(i) Every $(\in_\gamma\vee q_\delta,\in_\gamma\vee q_\delta)$-fuzzy hyperideal of H is an $(\in_\gamma,\in_\gamma\vee q_\delta)$-fuzzy hyperideal of H.
(ii) Every $(\in_\gamma,\in_\gamma)$-fuzzy hyperideal of H is an $(\in_\gamma,\in_\gamma\vee q_\delta)$-fuzzy hyperideal of H.

Proof. The proof is straightforward. □

Theorem 12. *Every $(\in_\gamma,\in_\gamma\vee q_\delta)$-fuzzy left (resp., right) hyperideal of H is an $(\in_\gamma,\in_\gamma\vee q_\delta)$-fuzzy bi-hyperideal of H.*

Proof. Let f be an $(\in_\gamma,\in_\gamma\vee q_\delta)$-fuzzy left hyperideal of H. Consider $\max\{\inf_{t\in(x\circ y)\circ z}f(t),\gamma\}\geq\min\{f(z),\delta\}$ by using the fact that f is an $(\in_\gamma,\in_\gamma\vee q_\delta)$-fuzzy left hyperideal of H. Also

$$\max\{\inf_{t\in(x\circ y)\circ z}f(t),\gamma\}=\max\{\inf_{t\in(z\circ y)\circ x}f(t),\gamma\}\geq\min\{f(x),\delta\}.$$

Combining the both, we have $\max\{\inf_{t\in(x\circ y)\circ z}f(t),\gamma\}\geq\min\{f(x),f(z),\delta\}$. Also if $y\leq x$, then $\max\{f(y),\gamma\}\geq\min\{f(y),\delta\}$. Hence f is an $(\in_\gamma,\in_\gamma\vee q_\delta)$-fuzzy bi-hyperideal of H. □

Theorem 13. *Let $\mathscr{F}(H)$ be the set of all $(\in_\gamma,\in_\gamma\vee q_\delta)$-fuzzy hyperideals of H. Then $(\mathscr{F}(H),\cup,\cap,\subseteq\vee_{q(\gamma,\delta)})$ forms the structure of a hyperlattice.*

Proof. (i) Reflexive: Since for all $f\in\mathscr{F}(H)$, $x_\alpha\in_\gamma f$ implies that $x_\alpha\in_\gamma\vee q_\delta f$, $\forall\, x\in H$. So $f\subseteq\vee_{q(\gamma,\delta)}f$.

(ii) Antisymmetric: For any $f,g\in\mathscr{F}(H)$ such that $f\subseteq\vee_{q(\gamma,\delta)}g$ and $g\subseteq\vee_{q(\gamma,\delta)}f$, we have

$$((\max\{g(a),\gamma\}\geq\min\{f(a),\delta\},(\max\{f(a),\gamma\}\geq\min\{g(a),\delta\})),$$

where $\delta,\gamma\in(0,1]$ such that $\gamma<\delta$. We have $\max\{\min\{g(x),\delta\},\gamma\}=\max\{\min\{f(x),\delta\},\gamma\}$. Hence $f=\vee_{q(\gamma,\delta)}g$.

(iii) Transitive: Let $f,g,h\in\mathscr{F}(H)$ such that $f\subseteq\vee_{q(\gamma,\delta)}g$ and $g\subseteq\vee_{q(\gamma,\delta)}h$. Then $f\subseteq\vee_{q(\gamma,\delta)}h$ by Corollary 1. Thus $(\mathscr{F}(H),\subseteq\vee_{q(\gamma,\delta)})$ is a poset.

Now given $f,g\in\mathscr{F}(H)$, we define $\inf\{f,g\}=f\cap g=\{\langle x,\min\{f(x),g(x)\}\rangle : x\in H\}$. In order to both $\inf\{f,g\}$ and $\sup\{f,g\}$ belong to $\mathscr{F}(H)$, we need to show that $f\cap g$ and $f\cup g$ are $(\in_\gamma,\in_\gamma\vee q_\delta)$-fuzzy hyperideals. Since the intersection of two $(\in_\gamma,\in_\gamma\vee q_\delta)$-fuzzy hyperideals is an $(\in_\gamma,\in_\gamma\vee q_\delta)$-fuzzy hyperideal,

$\inf\{f,g\} = f \cap g \in \mathscr{F}(H)$. Since

$$\begin{aligned}\max\{\inf_{z\in x\circ y} f(z),\gamma\} &= \max\{\inf_{z\in x\circ y}\{\max\{f(z),g(z)\},\gamma\}\} \\ &= \max\{\{\inf_{z\in x\circ y} f(z),\gamma\},\{\inf_{z\in x\circ y} g(z),\gamma\}\} \\ &\geq \max\{\{\min\{f(x),\delta\}\},\{\min\{g(x),\delta\}\}\} \\ &= \min\{f(x),\delta\},\end{aligned}$$

we obtain that $\sup\{f,g\} = f \cup g$ is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy right hyperideal of H. Similarly we can show that it is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy left hyperideal of H. Hence $\sup\{f,g\} = f \cup g$ is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy hyperideal of H. Thus $\sup\{f,g\} = f \cup g \in \mathscr{F}(H)$. Hence $(\mathscr{F}(H), \cup, \cap, \subseteq \vee_{q(\gamma,\delta)})$ forms a hyperlattice. □

Lemma 4. *Let H be an ordered LA-semihypergroup. If f and g are an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy right (resp., left) hyperideal of H, then $f \circ g$ is also an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy right (resp., left) hyperideal of H.*

Proof. The proof is straightforward. □

Theorem 14. *Let $\mathscr{F}(H)$ be the set of all $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy hyperideals of H. Then $(\mathscr{F}(H), \circ)$ forms an ordered LA-semihypergroup.*

Proof. The proof is straightforward. □

Proposition 2. *Let H be an ordered LA-semihypergroup with pure left identity e and if f is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy right hyperideal of H, then $f \circ f$ is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy hyperideal of H.*

Proof. By Theorem 9 every $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy right hyperideal of H is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy left hyperideal of H. Hence f is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy hyperideal of H. Assume that there do not exist some $x, y \in H$ such that $a \in x \circ y$ for $a \in H$. Then $f(a) = 0$. So $(\max\{\inf_{t\in x\circ y} f(t), \gamma\} \geq \min\{f(x), \delta\})$. Now if there exist $x, y \in H$ such that $a \in x \circ y$, then $\min\{f(a), \delta\} = \min\{\sup_{a\in x\circ y}\{\min\{f(x), f(y)\}\}, \delta\}$. If $a \in x \circ y$, then $a \circ b \in (x \circ y) \circ b = (b \circ y) \circ x$. Therefore

$$\begin{aligned}\min\{f(a),\delta\} &= \min\{\sup_{a\in x\circ y}\{\min\{f(x),f(y)\}\},\delta\} = \min\{\sup_{a\in x\circ y}\{\min\{f(y),f(x)\}\},\delta\} \\ &\leq \min\{\sup_{a\in x\circ y}\{\min\{f(b\circ y),f(x)\}\},\gamma\} \leq \min\{\sup_{a\circ b\subseteq (b\circ y)\circ x}\{\min\{f(b\circ y),f(x)\}\},\gamma\} \\ &\leq \max\{\inf_{z\in a\circ b} f(z),\gamma\}.\end{aligned}$$

Thus $(\max\{\inf_{z\in a\circ b} f(z), \gamma\} \geq \min\{f(a), \delta\})$. Also if $y \leq x$, then $\max\{f(y), \gamma\} \geq \min\{f(y), \delta\}$. Hence $f \circ f$ is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy right hyperideal of H. Now by Theorem 9 every $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy right hyperideal of H is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy left hyperideal of H. Hence $f \circ f$ is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy hyperideal of H. □

Theorem 15. *Let $\mathscr{F}(H)$ be the set of all $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy hyperideals of H and H have the pure left identity. Then for any $f, g, h \in \mathscr{F}(H)$, $f \circ (g \circ h) = g \circ (f \circ h)$.*

Proof. The proof is straightforward. □

Definition 20. *An $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy bi-hyperideal f of H is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy normal bi-hyperideal if $f(0) = 1$.*

Theorem 16. *Let f^* be an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy subset in H defined by $f(x) = f(x) + 1 - f(0)$. If f is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy bi-hyperideal of H, then f^* is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy normal bi-hyperideal of H which contains f.*

Proof. We have $f(0) = f(0)+1-f(0) = 1$. Given $x,y,z \in H$, we have

$$\begin{aligned}\min\{\inf_{z\in x\circ y} f(z), \gamma\} &= \min\{\inf_{z\in x\circ y} f(z)+1-f(0), \gamma\}\\ &\geq \max\{f(x)+1-f(0), f(y)+1-f(0), \delta\}\\ &= \max\{f(x), f(y), \delta\},\end{aligned}$$

and if $y \leq x$, then $\max\{f(y), \gamma\} \geq \min\{f(y), \delta\}$. Therefore f^* is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy normal bi-hyperideal of H. It is obvious that f^* contains f. □

Theorem 17. *Let f be an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy bi-hyperideal of H. Let $f^1 : [0,1] \to [0,1]$ and $f^2 = [0,1] \to [0,1]$ be increasing functions. Then the fuzzy subset f_f defined by $f(x) = f^1(f(x))$ is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy bi-hyperideal of H. In particular, if $f^1(f(0)) = 1$, then f_f is normal.*

Proof. Let $x,y \in H$. Then consider

$$\begin{aligned}\max\{\inf_{t\in x\circ y} f(t), \gamma\} &= \max\{\inf_{t\in x\circ y} f^1(f(t)), \gamma\}\\ &\geq \min\{f^1(f(x)), f^1(f(y)), \delta\}\\ &= \min\{f(x), f(y), \delta\},\end{aligned}$$

and if $y \leq x$, then $\max\{f(x), \gamma\} \geq \min\{f(y), \delta\}$. Thus f_f is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy bi-hyperideal of H. Now if $f^1(f(0)) = 1$, then $f(0) = 1$, so f_f is normal. □

4. IMAGES AND PREIMAGES OF $(\in_\gamma, \in_\gamma \vee q_\delta)$-FUZZY HYPERIDEALS

In this section we will present some results on images and preimages of $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy hyperideals of ordered LA-semihypergroups.

Definition 21. *A map $f : H_1 \to H_2$ where both H_1 and H_2 are LA-semihypergroups is called inclusion homomorphism if $f(a \circ b) \subseteq f(a) \circ f(b)$ for all $a,b \in H_1$.*

Let us denote by $\mathscr{F}(H_1)$ the family of fuzzy subsets in a set H_1. Let H_1 and H_2 be given classical sets. A mapping $h : H_1 \to H_2$ induces two mappings $\mathscr{F}_h : \mathscr{F}(H_1) \to \mathscr{F}(H_2)$, $f \mapsto \mathscr{F}_h(f)$, and $\mathscr{F}_h^{-1} : \mathscr{F}(H_2) \to \mathscr{F}(H_1)$, $g \mapsto \mathscr{F}_h^{-1}(g)$, where $\mathscr{F}_h(f)$ is given by

$$\mathscr{F}_h(f)(y) = \begin{cases} \sup_{y\in h(x)} f(x) & \text{if } h^{-1}(y) \neq \emptyset \\ 0 & \text{otherwise} \end{cases},$$

for all $y \in H_2$ and $\mathscr{F}_h^{-1}(g)$ is defined by $\mathscr{F}_h^{-1}(g)(x) = g(h(x))$. Then the mapping $\mathscr{F}_h$ (resp., $\mathscr{F}_h^{-1}$) is called an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy transformation (resp., inverse $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy transformation) induced by h. An $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy subset in H_1 has the $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy *property* if for any subset T of H_1 there exists $x_0 \in T$ such that $f(x_0) = \sup_{x\in T} f(x)$.

Theorem 18. *For a hyperhomomorphism $h : H_1 \to H_2$ of ordered LA-semihypergroups, let $\mathscr{F}_h : \mathscr{F}(H_1) \to \mathscr{F}(H_2)$ and $\mathscr{F}_h^{-1} : \mathscr{F}(H_2) \to \mathscr{F}(H_1)$ be an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy transformation and inverse $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy transformation, respectively, induced by h.*

(i) If $f \in \mathscr{F}(H_1)$ is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy LA-subsemihypergroup of H_1 which has the $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy property, then $\mathscr{F}_h(f)$ is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy LA-subsemihypergroup of H_2.

(ii) If $g \in \mathscr{F}(H_2)$ is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy LA-subsemihypergroup of H_2, then $\mathscr{F}_h^{-1}(g)$ is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy LA-subsemihypergroup of H_1.

Proof. (i) Given $h(x), h(y) \in h(H_1)$, let $x_0 \in h^{-1}(h(x))$ and $y_0 \in h^{-1}(h(y))$ be such that

$$f(x_0) = \sup_{a \in h^{-1}(h(x))} f(a).$$

Then

$$\begin{aligned}
\max\{F_h(f)(h(x)h(y)), \gamma\} &= \max\{\sup_{z \in h^{-1}(h(x)h(y))} (f)(z), \gamma\} \\
&\geq \max\{(f)(x_0 \circ y_0), \gamma\} \\
&\geq \min\{(f)(x_0), (f)(y_0), \delta\} \\
&= \min\{\sup_{a \in h^{-1}(h(x))} f(a), \sup_{b \in h^{-1}(h(y))} f(b), \delta\} \\
&= \min\{F_h(f)(h(x)), F_h(f)(h(y)), \delta\},
\end{aligned}$$

also if $y \leq x$ and we have $\max\{F_h(f)/(h(x)), \gamma\} \geq \min\{F_h(f)(h(y)), \delta\}$. Thus $F_h(f)$ is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy LA-subsemihypergroup of H_2.

(ii) For any $x, y \in H_1$, we have

$$\begin{aligned}
\max\{F_h^{-1}(g)(x \circ y), \gamma\} &= \max\{g(h)(xy), \gamma\} \\
&= \max\{g((h)(x)(h)(y)), \gamma\} \\
&\geq \min\{(g)(h(x)), (g)(h(y)), \delta\} \\
&= \min\{F_h^{-1}(g)(x), F_h^{-1}(g)(y), \delta\},
\end{aligned}$$

also if $y \leq x$ and we have $\max\{F_h^{-1}(g)(x), \gamma\} \geq \min\{F_h^{-1}(g)(y), \delta\}$. Hence $F_h^{-1}(g)$ is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy LA-subsemihypergroup of H_1. □

Theorem 19. *For a hyperhomomorphism $h : H_1 \to H_2$ of LA-semihypergroups, let $F_h : F(H_1) \to F(H_2)$ and $F_h^{-1} : F(H_2) \to F(H_1)$ be the $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy transformation and inverse $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy transformation, respectively, induced by h.*

(i) If $f \in F(H_1)$ is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy left (resp., right) ideal of H_1 which has the $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy property, then $F_h(f)$ is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy left (resp., right) ideal of H_2.

(ii) If $g \in F(H_2)$ is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy left (resp., right) ideal of H_2, then $F_h^{-1}(g)$ is an $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy left (resp., right) ideal of H_1.

Proof. The proof is straightforward. □

5. CONCLUSION

In this paper we introduced a new type of fuzzy subsets, namely $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy subsets in non-associative ordered semihypergroups. We defined different types of $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy hyperideals of ordered LA-semihypergroups. In future we are aiming to get more results related to

1. $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy subsets in regular and intra-regular ordered LA-semihypergroups,
2. $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy interior hyperideals in ordered LA-semihypergroups,
3. $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy quasi-hyperideals in ordered LA-semihypergroups.

ACKNOWLEDGEMENTS

The authors are highly grateful to referees for their valuable comments. The publication costs of this article were partially covered by the Estonian Academy of Sciences.

REFERENCES

1. Marty, F. Sur une generalization de la notion de groupe. In *8iem Congres Mathématiciens Scandinaves. Stockholm.*, 1934, 45–49.
2. Corsini, P. and Leoreanu, V. *Applications of Hyperstructure Theory*. Kluwer Academic Publications, 2003.
3. Davvaz, D. and Fotea, V. L. *Hyperring Theory and Applications*. International Academic Press, USA, 2007.
4. Bonansinga, P. and Corsini, P. On semihypergroup and hypergroup homomorphisms. *Boll. Un. Mat. Ital.*, 1982, **6**, 717–727.
5. Corsini, P. and Cristea, I. Fuzzy sets and non complete 1-hypergroups. *An. Sti. U. Ovid. Co-Mat.*, 2005, **13**, 27–54.
6. Davvaz, B. Some results on congruences on semihypergroups. *Bull. Malays. Math. Sci. Soc.*, 2000, **23**, 53–58.
7. Hasankhani, A. Ideals in a semihypergroup and Green's relations. *Ratio Math.*, 1999, **13**, 29–36.
8. Hila, K. and Dine, J. On hyperideals in left almost semihypergroups. *ISRN Algebra*, 2011, Article ID 953124, 8 pages.
9. Yaqoob, N., Corsini, P., and Yousafzai, F. On intra-regular left almost semihypergroups with pure left identity. *J. Math.*, 2013, Article ID 510790, 10 pages.
10. Yousafzai, F. and Corsini, P. Some characterization problems in LA-semihypergroups. *J. Algebra, Numb. Th. Adv. Appl.*, 2013, **10**, 41–55.
11. Heidari, D. and Davvaz, B. On ordered hyperstructures. *U.P.B. Sci. Bull. Series A*, 2011, **73**, 85–96.
12. Yaqoob, N. and Gulistan, M. Partially ordered left almost semihypergroups. *J. Egyptian Math. Soc.*, 2015, **23**, 231–235.
13. Zadeh, L. A. Fuzzy sets. *Inform. Control*, 1965, **8**, 338–353.
14. Murali, V. Fuzzy points of equivalent fuzzy subsets. *Inform. Sci.*, 2004, **158**, 277–288.
15. Pu, P. M. and Liu, Y. M. Fuzzy topology I, neighborhood structure of a fuzzy point and Moore-Smith convergence. *J. Math. Anal. Appl.*, 1980, **76**, 571–599.
16. Bhakat, S. K. and Das, P. $(\in, \in \vee q)$-fuzzy subgroups. *Fuzzy Sets Syst.*, 1996, **80**, 359–368.
17. Pibaljommee, B., Wannatong, K., and Davvaz, B. An investigation on fuzzy hyperideals of ordered semihypergroups. *Quasigroups Relat. Syst.*, 2015, **23**, 297–308.
18. Tang, J., Khan, A., and Luo, Y. F. Characterizations of semisimple ordered semihypergroups in terms of fuzzy hyperideals. *J. Intell. Fuzzy Syst.*, 2016, **30**, 1735–1753.
19. Azhar, M., Gulistan, M., Yaqoob, N., and Kadry, S. On fuzzy ordered LA-semihypergroups. *Int. J. Anal. Appl.*, 2018, **16**, 276–289.
20. Azhar, M., Yaqoob, N., Gulistan, M., and Khalaf, M. On $(\in, \in \vee q_k)$-fuzzy hyperideals in ordered LA-semihypergroups. *Disc. Dyn. Nat. Soc.*, 2018, Article ID 9494072, 13 pages.
21. Shabir, M. and Mahmood, T. Semihypergroups characterized by $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy hyperideals. *J. Intell. Fuzzy Syst.*, 2015, **28**, 2667–2678.
22. Shabir, M., Jun, Y. B., and Nawaz, Y. Semigroups characterized by $(\in_\gamma, \in_\gamma \vee q_k)$-fuzzy ideals. *Comput. Math. Appl.*, 2010, **60**, 1473–1493.
23. Rehman, N. and Shabir, M. Some characterizations of ternary semigroups by the properties of their $(\in_\gamma, \in_\gamma \vee q_\delta)$-fuzzy ideals. *J. Intell. Fuzzy Syst.*, 2014, **26**, 2107–2117.