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MATHEMATICS

Intuitionistic fuzzy-γ-retracts and interval-valued intuitionistic almost (near) compactness

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Abstract. The aim of this paper is to introduce the concepts of an intuitionistic fuzzy-γ-retract and an intuitionistic fuzzy-R-retract. Some characterizations of these new concepts are presented. Examples are given, and properties are established. Also, we study the concepts of interval-valued intuitionistic almost (near) compactness and define S_1-regular spaces. We prove that if an intuitionistic fuzzy topological space is an S_1-regular space and interval-valued intuitionistic almost (near) compact, then it is interval-valued intuitionistic compact.

Key words: intuitionistic fuzzy-γ-retract, S_1-regular space, interval-valued intuitionistic almost (near) compactness.

1. INTRODUCTION AND PRELIMINARIES

The concept of fuzzy sets was first proposed by Zadeh in 1965 [8]. This concept has a wide range of applications in various fields such as computer engineering, artificial intelligence, control engineering, operation research, management science, robotics, and many more. It gives us a tool to model the uncertainty present in a phenomenon that does not have sharp boundaries. Many papers on fuzzy sets have been published, showing their importance and applications to set theory, algebra, real analysis, measure theory, topology, etc.

Atanassov [1] extends the fuzzy set characterized by a membership function to the intuitionistic fuzzy set (IFS), which is characterized by a membership function, a non-membership function, and a hesitancy function. As a result, the IFS can describe the fuzzy characters of things in more detail and more comprehensively, which is found to be more effective in dealing with vagueness and uncertainty. Over the last few decades, the IFS theory has been receiving more and more attention from researchers and practitioners, and has been applied to various fields, including decision making, logic programming, medical diagnosis, pattern recognition, robotic systems, fuzzy topology, machine learning, and market prediction.

Intuitionistic fuzzy sets as a generalization of fuzzy sets can be useful in situations when the description of a problem by a (fuzzy) linguistic variable, given in terms of a membership function only, seems too rough. For example, in decision-making problems, particularly in the case of medical diagnosis, sales

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analysis, new product marketing, etc., there is a fair chance of the existence of a non-null hesitation part at each moment of evaluation of an unknown object. The concept of an intuitionistic fuzzy set, originally proposed by Atanassov [1], is an important tool for dealing with imperfect and imprecise information. Compared with Zadeh's fuzzy sets, an intuitionistic fuzzy set gives the membership and non-membership degree to which an element belongs to a set. Hence, coping with imperfect and imprecise information is more flexible and effective for intuitionistic fuzzy sets. In recent years, intuitionistic fuzzy set theory has been successfully applied in many practical fields, such as decision analysis and pattern recognition. Combining intuitionistic fuzzy set theory and rough set theory may be a promising topic that deserves further investigation. Some research has already been carried out on this topic.

In 1965, Zadeh presented the idea of a fuzzy set [8] as a means to represent uncertainty. This notion was originally introduced as a method to consider imprecision and ambiguity occurring in human discourse and thought. Many works by the same author and his colleagues appeared in the literature [3,4]. Later, topological structures in fuzzy topological spaces [5] were generalized to intuitionistic fuzzy topological spaces by Coker in [4], who then introduced the concept of an intuitionistic set [4]. This concept is the discrete form of an intuitionistic fuzzy set, and it is one of several ways of introducing vagueness in mathematical objects. On the other hand, the concept of a fuzzy retract was introduced by Rodabaugh [7].

The purpose of this paper is to construct the idea of intuitionistic fuzzy retracts, called IF-R-retracts, which use the generalization of intuitionistic fuzzy continuity. After giving the fundamental examples, we introduce the concepts of interval-valued intuitionistic almost (near) compactness and S_1-regular spaces and prove that if an intuitionistic fuzzy topological space (X,δ) is an S_1-regular space and interval-valued intuitionistic almost (near) compact, then it is an interval-valued intuitionistic compact.

Throughout this paper, X denotes a non-empty set. A fuzzy set in X is a function with domain X and values in I. The words intuitionistic fuzzy set and intuitionistic fuzzy topological space will be abbreviated as IF-set and IF-ts, respectively. Also, by $I(v)$, $C(v)$, and v' we will denote respectively the interior, closure, and the complement of an IF-set v. A mapping $r:(X,\delta)\to(Y,\gamma)$ is IF-continuous if $\forall\, v\in\gamma, r^{\leftarrow}(v)\in\delta$. Let (X,δ) be an IF-ts and $A\subseteq X$. Then a maximal subspace (A,δ_A) of (X,δ) is an IF-ts and is defined by $\delta_A=\{A\cap v : v\in\delta\}$.

Definition 1.1 [1]**.** *Let X be a nonempty set. An IF-set A is an object of the form* $A=\{x,\mu_A(x),\nu_A(x):x\in X\}$, *where the functions* $\mu_A:X\to[0,1]$ *and* $\nu_A:X\to[0,1]$ *denote, respectively, the degree of membership function (namely* $\mu_A(x)$*) and the degree of non-membership function (namely* $\nu_A(x)$*) of A,* $0\le\mu_A(x)+\nu_A(x)\le 1$, *for each* $x\in X$.

Remark 1.1 [2]**.** Atanassova and Doukovska introduced the following interesting geometrical interpretations to express an IF-set (see Fig. 1).

Definition 1.2 [1]**.** *Let X be a nonempty fixed set, and let I be the closed unit interval* [0,1]. *Consider two IF-sets* $A=\{x,\mu_A(x),\nu_A(x):x\in X\}$ *and* $B=\{x,\mu_B(x),\nu_B(x):x\in X\}$. *Then*
(i) $A'=\{x,\nu_A(x),\mu_A(x):x\in X\}$,
(ii) $A\le B\Leftrightarrow(\mu_A(x)\le\mu_B(x)$ *and* $\nu_A(x)\ge\nu_B(x))$, *for each* $x\in X$,
(iii) $A=B\Leftrightarrow A\le B$ *and* $B\le A$,
(iv) $\wedge A=\{x,\wedge\mu_A(x),\vee\nu_A(x)\}$, *and*
(v) $\vee A=\{x,\vee\mu_A(x),\wedge\nu_A(x)\}$.

Definition 1.3 [6]**.** *Let A be an IF-set of an IF-ts* (X,δ). *Then A is called*
(i) *an IF-regular open set if* $A=I(C(A))$,
(ii) *an IF-semi-open set if* $A\le C(I(A))$, $B\Leftrightarrow A\le B$ *and* $B\le A$,
(iii) *an IF-preopen set if* $A\le I(C(A))$,
(iv) *an IF-strongly semi-open set if* $A\le I(C(I(A)))$, *and*
(v) *an IF-semi-preopen set if* $A\le C(I(C(A)))$.
Their complements are called IF-regular closed, IF-semi-closed, IF-preclosed, IF-strongly semi-closed, and IF-semi-preclosed sets, respectively.

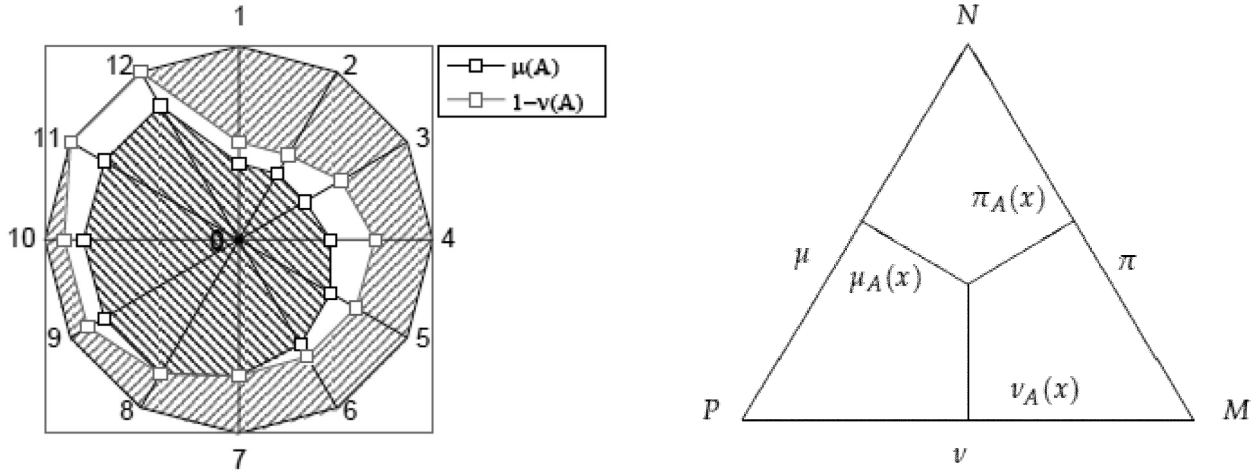

Fig. 1. Geometrical interpretations of an intuitionistic fuzzy set.

Definition 1.4. *Let f: $(X,\delta)\to(Y,\gamma)$ be a mapping from an IF-ts (X,δ) to another IF-ts (Y,γ). Then f is called*

(vi) *an IF-semi-continuous mapping if for each $v\in\gamma$ we have $f^{\leftarrow}(v)$ is an IF-semi-open set of X;*

(vii) *an IF-precontinuous mapping if for each $v\in\gamma$ we have $f^{\leftarrow}(v)$ is an IF-preopen set of X;*

(viii) *an IF-strongly semi-continuous mapping if for each $v\in\gamma$ we have $f^{\leftarrow}(v)$ is an IF-strongly semi-open set of X;*

(ix) *an IF-semi-precontinuous mapping if for each $v\in\gamma$ we have $f^{\leftarrow}(v)$ is an IF-semi-preopen set of X.*

Definition 1.5 [6]. *Let (X,δ) be an IF-ts and let $A\subset X$. Then the IF-subspace (A,δ_A) is called an IF-retract* (*IFR, for short*) of (X,δ) *if there exists an IF-continuous mapping $r:(X,\delta)\to(A,\delta_A)$ such that $r(a)=a$ for all $a\in A$. In this case r is called an IF-retraction.*

Definition 1.6 [6]. *Let (X,δ) be an IF-ts. Then (A,δ_A) is said to be an IF-neighbourhood retract (IF-nbd R, for short) of (X,δ) if (A,δ_A) is an IF-retract of (Y,δ_y) such that $A\subset Y\subset X, 1_Y\in\delta$.*

Definition 1.7 [6]. *Let (X,δ) be an IF-ts and $A\subset X$. Then the IF-subspace (A,δ_A) is called an IF-semi-retract* (*IFSR, for short*) (*respectively, IF-preretract, IF-strongly semi-retract, and IF-semi-preretract*) (*resp., IFPreR, IFSSR, IFSPR, for short*) *of (X,δ) if there exists an IF-semicontinuous* (*resp., IF-pre-continuous, IF-strongly semi-continuous, IF-semi-precontinuous*) *mapping $r:(X,\delta)\to(A,\delta_A)$ such that $r(a)=a$ for all $a\in A$. In this cas*e, *f is called an IF-semi-retraction* (*resp*., *IF-preretraction, IF-strongly semi-retraction, IF-semi-preretraction*).

Definition 1.8 [6]. *Let (X,δ) be an IF-ts. Then (A,δ_A) is said to be an IF-neighbourhood semi-retract* (*IF-nbd SR, for short*) (*resp., IF-nbd preretract, IF-nbd strongly semi-retract, IF-nbd semi-preretract*) (*IF-nbd PreR, IF-nbd SSR, IF-nbd SPR, for short*) *of (X,δ) if (A,δ_A) is an IFSR* (*resp., IFPreR, IFSSR, IFSPR*) *of (Y,δ_Y) such that $A\subset Y\subset X, 1_Y\in\delta$.*

Definition 1.9 [6]. *Let $f:(X,\delta)\to(Y,\gamma)$ be a function from an interval-valued intuitionistic fuzzy topological space* (I_t^v, *for short*) (X,δ) *into an I_t^v (Y,γ). Then f is said to be interval-valued intuitionistic-almost open* (*resp., closed*) *iff for each interval-valued intuitionistic fuzzy regular open* (*resp*., *closed*) *set $v\in X, f(v)\in Y$.*

Definition 1.10 [4]. Let (X,δ) be an I_t^v.

(i) *A family $\{\lambda_j : j\in J\}$ of interval-valued intuitionistic fuzzy sets* (I_s^v, *for short*) *of X is called an interval-valued intuitionistic fuzzy open cover* (I_o^v, *for short*) *of X iff $\bigvee_{j\in J}\lambda_j=\tilde{1}$.*

(ii) *A finite subfamily of an I_o^v G of X which is also an I_o^v of X is called a finite subcover of G.*

(iii) *A family* $M=\{\lambda_j : j\in J\}$ *of* I_s^v *of X satisfies the finite intersection property (FIP, for short) iff every finite subfamily* $\{\lambda_{j1},\lambda_{j2},\lambda_{j3},\ldots,\lambda_{jn}\}$ *of M satisfies the condition* $\wedge_{j=(1,\ldots n)}\lambda_j \neq \tilde{0}$.

(iv) *An* I_t^v (X,δ) *is called interval-valued intuitionistic fuzzy compact* (I_c^v, *for short*) *iff every* I_o^v *has a finite subcover.*

2. INTUITIONISTIC FUZZY-*γ*-RETRACTS

In this section the basic concept of an intuitionistic fuzzy-*γ*-retract is introduced, and some characterizations are presented. Examples and properties are established. Also, the relations between these new concepts are explained.

Definition 2.1. *Let* (X,δ) *be an IF-ts and* $A\subseteq X$. *Then a maximal subspace* (A,δ_A) of (X,δ) *is called an IF-γ-retract of* (X,δ) *(IF-γ-R, for short) if there exists an IF-γ-continuous mapping f*: $(X,\delta)\to(A,\delta_A)$ *such that* $f(x)=x$ *for all* $x\in A$. *In this case f is called an IF-γ-retraction.*

Remark 2.1. From the above definitions one may notice that

IFR ⇒ IFSSR ⇒ IFSR and IFPreR ⇒ IF-*γ*-R ⇒ IFSPR.

Example 2.1. Let λ_1 and λ_2 be IF-sets on $X=\{a,b,c\}$ defined by

$$\lambda_1=\left\langle x,\left(\frac{a}{0.5},\frac{b}{0.3},\frac{c}{0.3}\right),\left(\frac{a}{0.4},\frac{b}{0.5},\frac{c}{0.5}\right)\right\rangle,$$
$$\lambda_2=\left\langle x,\left(\frac{a}{0.6},\frac{b}{0.4},\frac{c}{0.4}\right),\left(\frac{a}{0.2},\frac{b}{0.3},\frac{c}{0.3}\right)\right\rangle,$$

$\delta=\{\underline{0},\underline{1},\lambda_1,\lambda_2\}$, and $A=\{x,y\}\subset X, f(a)=x, f(b)=f(c)=y$. Then (A,δ_A) is an IF-strongly semi-retract of (X,δ) but not an IF-retract.

Example 2.2. Let λ be an IF-set on $X=\{a,b,c\}$ defined by

$$\lambda=\left\langle x,\left(\frac{a}{0.1},\frac{b}{0.2}\right),\left(\frac{a}{0.3},\frac{b}{0.3}\right)\right\rangle,$$

$\delta=\{\underline{0},\underline{1},\lambda\}$, and $A=\{x\}\subset X$. Then (A,δ_A) is an IF-pre-retract of (X,δ) but not an IF-strongly semi-retract.

Example 2.3. Let λ be an IF-set on $X=\{a,b,c\}$ defined by

$$\lambda=\left\langle x,\left(\frac{a}{0.2},\frac{b}{0.3},\frac{c}{0.4}\right),\left(\frac{a}{0.2},\frac{b}{0.4},\frac{c}{0.4}\right)\right\rangle,$$

$\delta=\{\underline{0},\underline{1},\lambda\}$, and $A=\{x,y\}\subset X, f(a)=x, f(b)=f(c)=y$. Then (A,δ_A) is an IF-semi-retract of (X,δ) but not an IF-strongly semi-retract.

Example 2.4. Let λ be an IF-set on $X=\{a,b,c\}$ defined by

$$\lambda=\left\langle x,\left(\frac{a}{0.2},\frac{b}{0.3},\frac{c}{0.3}\right),\left(\frac{a}{0.2},\frac{b}{0.31},\frac{c}{0.31}\right)\right\rangle,$$

$\delta=\{\underline{0},\underline{1},\lambda\}$, and $A=\{x,y\}\subset X, f(a)=x, f(b)=f(c)=y$. Then (A,δ_A) is an IF-semi-preretract of (X,δ) but not an IF-*γ*-retract.

Example 2.5. Let λ be an IF-set on $X = \{a, b, c\}$ defined by

$$\lambda = \left\langle x, \left(\frac{a}{0.1}, \frac{b}{0.3}, \frac{c}{0.5}\right), \left(\frac{a}{0.2}, \frac{b}{0.4}, \frac{c}{0.5}\right)\right\rangle,$$

$\delta = \{\underline{0}, \underline{1}, \lambda\}$, and $A = \{x, y\} \subset X, f(a) = x, f(b) = f(c) = y$. Then (A, δ_A) is an IF-γ-retract of (X, δ) but not an IF-semi-retract.

Example 2.6. Let λ be an IF-set on $X = \{a, b, c\}$ defined by

$$\lambda = \left\langle x, \left(\frac{a}{0.3}, \frac{b}{0.5}, \frac{c}{0.2}\right), \left(\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.8}\right)\right\rangle,$$

$\delta = \{\underline{0}, \underline{1}, \lambda\}$, and $A = \{x, y\} \subset X, f(a) = x, f(b) = f(c) = y$. Then (A, δ_A) is an IF-γ-retract of (X, δ) but not an IF-preretract.

Remark 2.2. Let (X, δ) be an IF-ts and $Z \subset Y \subset X$. If (Y, δ_y) is an IF-γ-retract of (X, δ) and $(Z, (\delta_y)z)$ is an IF-γ-retract of (Y, δ_y), then $(Z, (\delta_y)z)$ need not be an IF-γ-retract of (X, δ).

Example 2.7. Let $X = \{a, b, c\}, Y = \{a, b\}, Z = \{a\}$, and let λ_1, λ_2 be IF-sets on X defined by

$$\lambda_1 = \left\langle x, \left(\frac{a}{0.2}, \frac{b}{0.3}, \frac{c}{0.3}\right), \left(\frac{a}{0.5}, \frac{b}{0.5}, \frac{c}{0.6}\right)\right\rangle,$$

$$\lambda_2 = \left\langle x, \left(\frac{a}{0.5}, \frac{b}{0.5}, \frac{c}{0.4}\right), \left(\frac{a}{0.5}, \frac{b}{0.5}, \frac{c}{0.6}\right)\right\rangle,$$

$\delta = \{\underline{0}, \underline{1}, \lambda_1, \lambda_2\}$. Then (Y, δ_y) is an IF-γ-retract of (X, δ), and $(Z, (\delta_y)z)$ is an IF-γ-retract of (Y, δ_y), but $(Z, (\delta_y)z)$ is not an IF-γ-retract of (X, δ).

Remark 2.3. Let $(X_1, \delta_1), (X_2, \delta_2), (Y_1, \sigma_1)$, and (Y_2, σ_2) be IF-ts's. If $f_1 : X_1 \to Y_1$ is IF-γ-continuous and $f_2 : X_2 \to Y_2$ is IF-γ-continuous, then the product $f_1 \times f_2 : X_1 \times X_2 \to Y_1 \times Y_2$ need not be IF-γ-continuous.

Example 2.8. Let $X_1 = Y_1 = \{a, b\}$, $\delta_1 = \{\underline{0}, \underline{1}, \lambda_1, \lambda_2\}$, and $\sigma_1 = \{\underline{0}, \underline{1}, \nu_1, \nu_2\}$. Let λ_1, λ_2 be IF-sets on X_1 and let ν_1, υ_2 be IF-sets on Y_1, defined by

$$\lambda_1 = \left\langle x, \left(\frac{a}{0.3}, \frac{b}{0.4}\right), \left(\frac{a}{0.3}, \frac{b}{0.2}\right)\right\rangle,$$

$$\lambda_2 = \left\langle x, \left(\frac{a}{0.1}, \frac{b}{0.1}\right), \left(\frac{a}{0.32}, \frac{b}{0.31}\right)\right\rangle,$$

$$\nu_1 = \left\langle x, \left(\frac{a}{0.3}, \frac{b}{0.7}\right), \left(\frac{a}{0.22}, \frac{b}{0.2}\right)\right\rangle,$$

$$\nu_2 = \left\langle x, \left(\frac{a}{0.2}, \frac{b}{0.11}\right), \left(\frac{a}{0.32}, \frac{b}{0.31}\right)\right\rangle,$$

and let $f_1 = id_{X_1} : X_1 \to Y_1$ be defined by $f_1(x) = X, \forall x \in X_1$. Then f_1 is IF-γ-continuous. Also, let $X_2 = Y_2 = \{x, y\}$, $\delta_2 = \{\underline{0}, \underline{1}, \lambda\}$, and $\sigma_2 = \{\underline{0}, \underline{1}, \nu\}$, Let λ be an IF-set on X_2 and let ν_1 be an IF-set on Y_2, defined by

$$\lambda = \left\langle x, \left(\frac{a}{0.4}, \frac{b}{0.5}\right), \left(\frac{a}{0.2}, \frac{b}{0.12}\right)\right\rangle,$$

$$\nu = \left\langle x, \left(\frac{a}{0.1}, \frac{b}{0.22}\right), \left(\frac{a}{0.32}, \frac{b}{0.31}\right)\right\rangle,$$

and let $f_2 = id_{X_2} : X_2 \to Y_2$ be defined by $f_2(x) = X, \forall x \in X_2$. Then f_2 is IF-γ-continuous, but $f_1 \times f_2$ need not be IF-γ-continuous.

Remark 2.4. Let (X,δ) and (Y,y) be IF-ts's and $A \subset X, B \subset Y$. If (A,δ_A) is an IF-γ-retract of (X,δ) and (B,y_B), is an IF-γ-retract of (Y,σ), then $(A\times B,(\delta \times y)_{A\times B})$ need not to be an IF-γ-retract of $(X\times Y, \delta\times\sigma)$.

Remark 2.5. IF-semi-retracts and IF-preretracts are independent concepts.

Example 2.9. Let λ_1 and λ_2 be IF-sets on $X = \{\mathrm{a,b,c}\}$ defined by

$$\lambda_1 = \left\langle x, \left(\frac{a}{0.1}, \frac{b}{0.2}, \frac{c}{0.3}\right), \left(\frac{a}{0.4}, \frac{b}{0.6}, \frac{c}{0.6}\right)\right\rangle,$$

$$\lambda_2 = \left\langle x, \left(\frac{a}{0.2}, \frac{b}{0.3}, \frac{c}{0.4}\right), \left(\frac{a}{0.4}, \frac{b}{0.4}, \frac{c}{0.4}\right)\right\rangle,$$

$\delta = \{\underline{0}, \underline{1}, \lambda_1, \lambda_2\}$, and $A = \{x, y\} \subset X, f(a) = x, f(b) = f(c) = y$. Then (A,δ_A) is an IF-semi-retract of (X,δ) but not an IF-preretract.

Example 2.10. Let λ be an IF-set on $X=\{a,b,c\}$ defined by

$$\lambda = \left\langle x, \left(\frac{a}{0.1}, \frac{b}{0.3}, \frac{c}{0.5}\right), \left(\frac{a}{0.1}, \frac{b}{0.4}, \frac{c}{0.2}\right)\right\rangle,$$

$\delta = \{\underline{0}, \underline{1}, \lambda\}$, and $A = \{x, y\} \subset X, f(a) = x, f(b) = f(c) = y$. Then (A,δ_A) is an IF-preretract of (X,δ) but not an IF-semi-retract.

3. IF-R-CONTINUITY AND IF-R-RETRACTS

In this section the basic concepts of intuitionistic fuzzy perfectly retracts, intuitionistic fuzzy R retracts, and intuitionistic fuzzy completely retracts and some characterizations are discussed. Many examples are given, and some properties are established. Also, we define the relations between these new concepts.

Definition 3.1. *Let $f: (X,\delta) \to (Y,\gamma)$ be a mapping from an IF-ts (X,δ) to another IF-ts (Y,γ). Then f is called*

(i) *an IF-perfectly continuous (IFPC, for short) mapping if for each $\nu \in \gamma$ we have $f^{\leftarrow}(\nu)$ is both an IF-open and an IF-closed set of X,*

(ii) *an IF-completely continuous (IFCC, for short) mapping if for each $\nu \in \gamma$ we have $f^{\leftarrow}(\nu)$ is an IF-regular open set of X,*

(iii) *an IF-R-continuous (IFRC, for short) mapping if for each IF-regular open $\nu \in \gamma$ we have $f^{\leftarrow}(\nu)$ is an IF-regular open set of X.*

Remark 3.1. The implications between these different concepts are given by the following diagram:

IFPC $\Rightarrow$ IFCC $\Rightarrow$ IFRC.

The converses of the above implications need not be true in general, as shown by the following examples.

Example 3.1. Let $X=\{a,b\}$, $Y=\{1,2\}$, Let (X,δ) and (Y,γ) be two IF-ts's where $\delta=\{0,1,\alpha,\beta\}$, and $\gamma=\{0,1,\theta_1,\theta_2\},\alpha,\beta,\theta_1$ and θ_2 are defined by

$$\beta=\left\langle x,\left(\frac{a}{0.3},\frac{b}{0.2}\right),\left(\frac{a}{0.2},\frac{b}{0.1}\right)\right\rangle,$$
$$\alpha=\left\langle x,\left(\frac{a}{0.2},\frac{b}{0.1}\right),\left(\frac{a}{0.3},\frac{b}{0.2}\right)\right\rangle,$$
$$\theta_1=\left\langle x,\left(\frac{1}{0.2},\frac{2}{0.3}\right),\left(\frac{1}{0.1},\frac{2}{0.2}\right)\right\rangle,$$
$$\theta_2=\left\langle x,\left(\frac{1}{0.1},\frac{2}{0.2}\right),\left(\frac{1}{0.2},\frac{2}{0.3}\right)\right\rangle,$$

$f(a)=2$, $f(b)=1$. Then f is IFRC but not IFCC.

Example 3.2. Let $X=Y=[0,1]$. Let (X,δ) and (Y,η) be two IF-ts's where $\delta=\left\{0,1,C_{0.7,0.2},C_{\alpha,\beta}:0\le\alpha\le\frac{1}{4},0\le\beta\le\frac{1}{2}\right\}$ and $\eta=\left\{\underline{0},\underline{1},C_{\alpha,\beta}:0\le\alpha\le\frac{1}{4},0\le\beta\le\frac{1}{2}\right\}$, $f(x)=x$. Then $f:(X,\delta)\to(Y,\eta)$ is IFCC but not IFPC.

Definition 3.2. *An IF-ts (X,δ) is called an IF-extremely disconnected space (IFED-space, for short) if the closure of every IF-open set of X is an IF-open set.*

Lemma 3.1. *Let (X,δ) be an IFED-space. Then, if λ is an IF-regular open set of X, it is both an IF-open set and an IF-closed set.*

Proof. Let λ be an IF-regular open set of *X*, then $\lambda=I(C(\lambda))$ since every IF-regular open set is IF-open. Then λ is an IF-open set of *X* and because (X,δ) is an IFED-space, $C(\lambda)=\lambda$. Then λ is an IF-closed set. □

Theorem 3.1. *Let (X,δ) be an IFED-space, and let $f:(X,\delta)\to(Y,\gamma)$ be a mapping. Then the following are equivalent:*
(i) *f is IFPC,*
(ii) *f is IFCC.*

Proof. It follows from Lemma 3.1. □

Theorem 3.2. *Let $f:(X,\delta)\to(Y,\gamma)$ be a mapping. Then f is IFPC (resp., IFCC) iff the inverse image of every IF-closed set of Y is both an IF-open set and an IF-closed (resp., IF-regular open) set of X.*

Proof. Obvious. □

Theorem 3.3. *Let $f:(X,\delta)\to(Y,\gamma)$ be a mapping, and let $g:X\to X\times Y$ be its graph. If g is IFPC (resp., IFCC) so f is IFPC (resp., IFCC).*

Proof. Let λ be an IF-open set of *Y*. Then $1\times\lambda$ is an IF-open set of $X\times Y$. Since *g* is IF-perfectly continuous, $g^{\leftarrow}(1\times\lambda)$ is both an IF-open set and an IF-closed set of *X*. Then we have $g^{\leftarrow}(1\times\lambda)=1\wedge f^{\leftarrow}(\lambda)=f^{\leftarrow}(\lambda)$. Therefore $f^{\leftarrow}(\lambda)$ is both an IF-open set and an IF-closed set of *X*. Hence *f* is IFPC. The proof for IFCC is by the same fashion. □

Definition 3.3. *Let (X,δ) be an IF-ts, and let $A\subset X$. Then the IF-subspace (A,δ_A) is called an IF-perfectly retract (IFPR, for short) (resp., IF-completely retract, IFR-retract) (resp., IFCR, IFRR, for short) of (X,δ) if there exists an IFPC (resp., IFCC, IFRC) mapping $r:(X,\delta)\to(A,\delta_A)$ such that $r(a)=$ a for all a $\in A$. In this case r is called an IF-perfectly retraction (resp., IF-completely retraction, IF-R-retraction).*

Remark 3.1. The implications between these different concepts are given by the following diagram:

$$\text{IFPR} \Rightarrow \text{IFCR}.$$

The converse of the above implication need not be true in general, as shown by the following examples.

Example 3.3. Let λ_1 and λ_2 be IF-sets on $X = \{a, b\}$ defined by

$$\lambda_1 = \left\langle x, \left(\frac{a}{0.3}, \frac{b}{0.2}\right), \left(\frac{a}{0.3}, \frac{b}{0.5}\right)\right\rangle,$$

$$\lambda_2 = \left\langle x, \left(\frac{a}{0.3}, \frac{b}{0.3}\right), \left(\frac{a}{0.3}, \frac{b}{0.3}\right)\right\rangle,$$

$\delta = \{\underline{0}, \underline{1}, \lambda_1, \lambda_2\}$, and $A = \{a\} \subset X$. Then (A, δ_A) is an IFCR of (X, δ) but not an IFPR.

Theorem 3.4. *Let (X, δ) be an IF-ts, $A \subset X$ and let $r : (X, \delta) \to (A, \delta_A)$ be a mapping such that $r(a) = a$ for all $a \in A$. If the graph $g : (X, \delta) \to (X \times A, \theta)$ of r is IFPC (resp., IFCC), then f is an IF-retraction, where θ is the product topology generated by δ and δ_A.*

Proof. It follows directly from Theorem 3.3. □

Definition 3.4. *Let (X, δ) be an IF-ts. Then (A, δ_A) is said to be an IF-neighbourhood-perfectly retract (resp., IF-neighbourhood completely retract) (resp., IF-nbd PR, IF-nbd CR, for short) of (X, δ) if (A, δ_A) is an IFPR (resp., IFCR) of (Y, δ_y) such that $A \subset Y \subset X, 1_Y \in \delta$.*

Remark 3.2.
(i) Every IFPR is an IF-nbd PR, but the converse is not true.
(ii) Every IFCR is an IF-nbd CR, but the converse is not true.

Example 3.4. Let $X = \{a, b, c\}$, $A = \{a\} \subset \text{X}$, and let λ_1 and λ_2 be IF-sets on X defined by

$$\lambda_1 = \left\langle x, \left(\frac{a}{0.3}, \frac{b}{0.3}, \frac{c}{0.4}\right), \left(\frac{a}{0.3}, \frac{b}{0.3}, \frac{c}{0.1}\right)\right\rangle,$$

$$\lambda_2 = \left\langle x, \left(\frac{a}{1}, \frac{b}{1}, \frac{c}{0}\right), \left(\frac{a}{0}, \frac{b}{0}, \frac{c}{1}\right)\right\rangle.$$

Consider $\delta = \{\underline{0}, \underline{1}, \lambda_1, \lambda_2, \lambda_1 \vee \lambda_2, \lambda_1 \wedge \lambda_2\}$. Then (A, δ_A) is an IF-nbd CR of (X, δ) but not an IFCR of (X, δ), and it is an IF-nbd PR of (X, δ) but not an IFPR of (X, δ).

4. INTERVAL-VALUED INTUITIONISTIC COMPACTNESS

In this section we introduce the concepts of interval-valued intuitionistic almost (near) compactness and define S_1-regular spaces. We prove that if (X, δ) is an S_1-regular space and interval-valued intuitionistic almost (near) compact, then it is interval-valued intuitionistic compact.

Definition 4.1 [4]. *Let (X, δ) be an interval-valued intuitionistic fuzzy topological space (I_t^v for short).*
(i) *A family $\{\lambda_j : j \in J\}$ of interval-valued intuitionistic fuzzy open sets of X is called an I_o^v of X iff $\bigvee_{j \in J} \lambda_j = \tilde{1}$.*
(ii) *A finite subfamily of an I_o^v G of X that is also an I_o^v of X is called a finite subcover of G.*
(iii) *A family $M = \{\lambda_j : j \in J\}$ of I_o^v of X satisfies the finite intersection property iff every finite subfamily $\{\lambda_{j_1}, \ldots, \lambda_{j_n}\}$ of M satisfies the condition $\bigwedge_{i=(1,\ldots,n)} \lambda_{j_i} \neq \tilde{0}$.*
(iv) *An I_t^v is called I_c^v iff every I_o^v of X has a finite subcover.*

Definition 4.2.

(i) *An I_t^v is called interval-valued intuitionistic fuzzy almost compact ($I_{almost_c}^v$, for short) iff every I_o^v of X has a finite subcollection whose closures cover X.*

(ii) *An I_t^v is called interval-valued intuitionistic fuzzy nearly compact ($I_{nearly_c}^v$, for short) iff every I_o^v of X has a finite subcollection such that the interiors of closures of sets in this subcollection cover X.*

Example 4.1. Let $X = I$ and let $\{\lambda_i : i = 1,2,3,...\}$ be intuitionistic fuzzy sets defined as follows. First we define $\lambda_i = \langle x, \mu_{\lambda_i}, \nu_{\lambda_i} \rangle$ and $\lambda = \langle x, \mu_\lambda, \nu_\lambda \rangle$ by

$$\mu_{\lambda_i}(x) = \begin{cases} 0.8, & x = 0 \\ nx, & 0 < x < \dfrac{1}{n} \\ 1, & \dfrac{1}{n} < x \leq 1, \end{cases}$$

$$\nu_{\lambda_i}(x) = \begin{cases} 0.1, & x = 0 \\ 1 - nx, & 0 < x < \dfrac{1}{n} \\ 0, & \dfrac{1}{n} < x \leq 1, \end{cases}$$

$$\mu_\lambda(x) = \begin{cases} 0.8, & x = 0 \\ 1, & \text{otherwise}, \end{cases}$$

$$\nu_\lambda(x) = \begin{cases} 0.1, & x = 0 \\ 0, & \text{otherwise}. \end{cases}$$

Second, we define an intuitionistic fuzzy topological space as follows:
$\delta = \{\lambda \underline{0}, \underline{1}, \lambda_i, \lambda\}$. Since $\{\lambda_i : i = 1,2,3,...\}$ are IF-open sets of X and $\vee_{i \in J} \lambda_i = \tilde{1}$, then $\{\lambda_i : i = 1,2,3,...\}$ is an I_o^v of X. As λ is a finite subfamily of an I_o^v, then $\lambda \subseteq \lambda_i$ implies that λ is a finite subcover of X. Then the intuitionistic fuzzy topological space (X, δ) is an intuitionistic fuzzy compact space.

Theorem 4.1. *Let I_c^v be an interval-valued intuitionistic fuzzy compact space, let $I_{nearly_c}^v$ be an interval-valued intuitionistic fuzzy nearly compact space, and let $I_{almost_c}^v$ be an interval-valued intuitionistic fuzzy almost compact space. Then the implications between these different concepts are given by the following diagram*:

$$I_c^v \Rightarrow I_{nearly_c}^v \Rightarrow I_{almost_c}^v.$$

Proof. Let (X, δ) be an I_c^v space, and let $\{\lambda_i, i \in \Gamma\}$ be an I_o^v of X. Then

$$\vee_{i \in \Gamma} \lambda_i = \tilde{1} \Rightarrow \tilde{1} - \vee_{i \in \Gamma} \lambda_i(x) = \tilde{0} \Rightarrow \wedge_{i \in \Gamma} (\tilde{1} - \mu_{\lambda i}^U(x)) = \tilde{0} \Rightarrow \wedge \nu_{\lambda i}^U(x) = \tilde{0} \forall x \in X.$$

Then $\exists G_{k=(1,...,n)}$ is a finite subcover such that $G_{k(1,...,n)} \leq \lambda_i$. We have $G_{k=(1,...,n)} = I(G_{k=(1,...,n)})$, therefore

$$\vee G_{k=(1,...,n)} = I(\vee G_{k=(1,...,n)}) \leq CI(\vee G_{k(1,...,n)}) = \tilde{1},$$

and hence (X, δ) is $I_{nearly_c}^v$. For the second implication, assuming (X, δ) to be $I_{nearly_c}^v$, we obtain a finite subset $G_{k=(1,...,n)}$ such that $CI(\vee G_{k=(1,...,n)}) = \tilde{1}$, since

$$G_{k=(1,...,n)} = (G_{k=(1,...,n)}) \leq CI(G_{k=(1,...,n)}) \leq C(G_{k=(1,...,n)}) = \tilde{1}.$$

It is obvious that $\vee C(G_{k=(1,...,n)}) = \tilde{1}$. Hence, (X, δ) is $I_{almost_c}^v$. □

Theorem 4.2. *Let* $(X,\delta),(Y,y)$ *be interval-valued intuitionistic fuzzy-regular spaces. Let* $f:(X,\delta)\to(Y,\gamma)$ *be a surjection and interval-valued intuitionistic fuzzy-almost continuous. If* (X,δ) *is* $I^{v}_{almost_c}$, *then so is* (Y,γ).

Proof. Let $\{\lambda_i, i\in\Gamma\}$ be an I^{v}_{o} of *Y*. Then from the interval-valued intuitionistic fuzzy-almost continuity of *f* it follows that $\{f^{\leftarrow}IC(\lambda_i), i\in\Gamma\}$ is an I^{v}_{o} of *X*. Since (X,δ) is $I^{v}_{almost_c}$, there exists $(\lambda_{i=(1,\dots,n)})$ such that $\vee_{i=(1,\dots,n)}C(f^{\leftarrow}IC(\lambda_i))=\tilde{1}$. Hence

$$f(\vee_{i=(1,\dots,n)}C(f^{\leftarrow}IC(\lambda_i)))=\vee_{i=(1,\dots,n)}(f(C(f^{\leftarrow}IC(\lambda_i)))=f(\tilde{1})=\tilde{1}.$$

But since $IC(\lambda_i)\le C(\lambda_i)$ and from the interval-valued intuitionistic fuzzy-almost continuity of *f*, we see that $f^{\leftarrow}C(\lambda_i)$ must be an interval-valued intuitionistic fuzzy-almost continuous containing $(f^{\leftarrow}IC(\lambda_i))$ and hence $C(f^{\leftarrow}IC(\lambda))$. Thus

$$fC(f^{\leftarrow}IC(\lambda_i))\le f(f^{\leftarrow}C(\lambda_i))\le C(\lambda_i)$$

for each $i\in\Gamma$, which implies $\vee_{i\in\Gamma}C(\lambda_i)=\tilde{1}$. Hence (Y,y) is also $I^{v}_{almost_c}$. □

Theorem 4.3. *Let* $(X,\delta),(Y,y)$ *be interval-valued intuitionistic fuzzy topological spaces. Let* $f:(X,\delta)\to(Y,y)$ *be a surjection and interval-valued intuitionistic fuzzy-weakly continuous. If* (X,δ) *is an* I^{v}_{c}, *then* (Y,γ) *is* $I^{v}_{almost_c}$.

Proof. Let $\lambda=\{\lambda_i, i\in\Gamma\}$ be an I^{v}_{o} of *Y*. Since *f* is an IVIF-weakly continuous mapping, then we have $f^{\leftarrow}(\lambda_i)\le I(f^{\leftarrow}C(\lambda_i))$. Hence, $v=\{I(f^{\leftarrow}C(\lambda_i)), i\in\Gamma\}$ is an I^{v}_{o} of *X*. Since (X,δ) is an I^{v}_{c}, there exists a finite subcover of *v* indexed by $\lambda_{i(i=1,2,3,\dots,n)}$ such that $\vee_{i=(1,\dots,n)}C(f^{\leftarrow}IC(\lambda_i))=\tilde{1}$. Therefore,

$$f(\vee_{i=(1,\dots,n)}I(f^{\leftarrow}C(\lambda_i)))=\vee_{i=(1,\dots,n)}f(I(f^{\leftarrow}C(\lambda_i)))=f(\tilde{1})=\tilde{1}.$$

Now from

$$f(I(f^{\leftarrow}C(\lambda_i)))\le f(f^{\leftarrow}C(\lambda_i))=C(\lambda_i),\forall i\in\Gamma,$$

we deduce

$$f(I(f^{\leftarrow}C(\lambda_i)))\le C(\lambda_i),\forall i\in\Gamma.$$

Hence $\vee_{i\in\Gamma}C(\lambda_i)=\tilde{1}$, which implies (Y,y) is $I^{v}_{almost_c}$. □

Theorem 4.4. *Let* $(X,\delta),(Y,y)$ *be interval-valued intuitionistic fuzzy topological spaces. Let* $f:(X,\delta)\to(Y,\gamma)$ *be a surjection and interval-valued intuitionistic fuzzy-strongly continuous. If* (X,δ) *is an* $I^{v}_{almost_c}$, *then* (Y,γ) *is* I^{v}_{c}.

Proof. Let $\{\lambda_i, i\in\Gamma\}$ be an I^{v}_{o} of *Y*. Since *f* is IVIF-strongly continuous and hence a continuous mapping, then we have $\{f^{\leftarrow}(\lambda_i), i\in\Gamma\}$ is an I^{v}_{o} of *X*. Since (X,δ) is $I^{v}_{almost_c}$, there exists a finite subfamily $\lambda_{i(i=1,\dots,n)}$ such that $\vee_{i=(1,\dots,n)}C(f^{\leftarrow}(\lambda_i))=\tilde{1}$. From the surjectivity and fuzzy strong continuity of *f* we obtain

$$f(\vee_{i=(1,\dots,n)}C(f^{\leftarrow}(\lambda_i)))=\vee_{i=(1,\dots,n)}f(C(f^{\leftarrow}C(\lambda_i)))\le\vee_{i=(1,\dots,n)}f(f^{\leftarrow}(\lambda_i))=\vee_{i=(1,\dots,n)}\lambda_i=f(\tilde{1})=\tilde{1}.$$

Hence (Y,γ) is I^{v}_{c}. □

Theorem 4.5. *Let* (X,δ) *be an interval-valued intuitionistic fuzzy topological space. Then the following conditions are equivalent*:

(i) (X,δ) *is* $I^{v}_{almost_c}$.

(ii) *For every family* $\lambda=\{\lambda_i, i\in\Gamma\}$, *where* $\lambda_i=\left\langle x,\left(\mu^{L}_{\lambda_i},\mu^{U}_{\lambda_i}\right),\left(\nu^{L}_{\lambda_i},\nu^{U}_{\lambda_i}\right)\right\rangle$ *for all* $i\in\Gamma$, *of interval-valued intuitionistic fuzzy regular closed sets such that* $\wedge_{i\in\Gamma}\lambda_i=\tilde{0}$, *there exists a finite subfamily* $\lambda_{i(i=1,\dots,n)}$ *such that* $\wedge_{i=(1,\dots,n)}I(\lambda_i)=\tilde{0}$.

(iii) $\wedge_{i\in\Gamma} C(\lambda_i) \neq \tilde{0}$ *holds for each family* $\{\lambda_i, i\in\Gamma\}$ *of interval-valued intuitionistic fuzzy regular-regular open sets where* $\lambda_i = \langle x, (\mu^L_{\lambda_i}, \mu^U_{\lambda_i}), (\nu^L_{\lambda_i}, \nu^U_{\lambda_i})\rangle$ *for all* $i\in\Gamma$.

(iv) *Every interval-valued intuitionistic fuzzy-regular open cover of X contains a finite subfamily whose closures cover X.*

Proof. (i)$\Rightarrow$(ii) Let $\lambda = \{\langle x, (\mu^L_{\lambda_i}, \mu^U_{\lambda_i}), (\nu^L_{\lambda_i}, \nu^U_{\lambda_i})\rangle, i\in\Gamma\}$ be a family of interval-valued intuitionistic fuzzy regular-regular closed sets in X with $\wedge_{i\in\Gamma}\lambda_i = \tilde{0}$. Then $\vee_{i\in\Gamma}\lambda_i' = \tilde{1}$. Since $(\lambda_i)' = I(C(\lambda_i)')$, we have $\vee_{i\in\Gamma} I(C(\lambda_i)') = \tilde{1}$. Because (X,δ) is $I^v_{almost_c}$ it follows that there exists a finite subfamily $\lambda_{i=(1,\ldots,n)}$ of λ such that $\vee_{i=(1,\ldots,n)} C(I(C(\lambda_i)')) = \tilde{1}$, therefore

$$[\vee_{i=(1,\ldots,n)} C(I(C(\lambda_i)'))]' = \wedge_{i=(1,\ldots,n)}[C(I(C(\lambda_i)'))]' = \wedge_{i=(1,\ldots,n)} I(C(I(\lambda_i))) = \wedge_{i=(1,\ldots,n)} I(\lambda_i) = \tilde{0}.$$

(ii)$\Rightarrow$ (iii) Let $\{\lambda_i, i\in\Gamma\}$ be a family of interval-valued intuitionistic fuzzy regular-regular open sets, and suppose that $\wedge_{i\in\Gamma} C(\lambda_i) = \tilde{0}$. Since $\{C\lambda_i, i\in\Gamma\}$ is a family of interval-valued intuitionistic fuzzy regular-regular open sets, there exists a finite subfamily $\{C\lambda_{i=(1,\ldots,n)}\}$ such that $\wedge_{i=(1,\ldots,n)} I(C(\lambda_i)) = \wedge_{i=(1,\ldots,n)}\lambda_i = \tilde{0}$, which is a contradiction.

(iii)$\Rightarrow$ (iv) Let $\lambda = \{C\lambda_i, i\in\Gamma\}$ be a family of interval-valued intuitionistic fuzzy regular-regular open sets covering X. Suppose that $\vee_{i=(1,\ldots,n)} C(\lambda_i) \neq \tilde{1}$ for each finite subcover of λ. Then $\{[C(\lambda_i)]' : i\in\Gamma\}$ is a family of IVIF-regular open sets, since $[I(C([C(\lambda_i)]')] = [I(I([C(\lambda_i)]')] = I(\lambda_i)' = [C(\lambda_i)]'$. Hence $\wedge_{i\in\Gamma} C[C(\lambda_i)]' \neq \tilde{0} \Rightarrow \wedge_{i\in\Gamma}[I(C(\lambda_i)]' \neq \tilde{0}$, which is in contradiction with $\vee_{i\in\Gamma}\lambda_i = 1$.

(iv)$\Rightarrow$ (i) Obvious, since every interval-valued intuitionistic fuzzy regular-regular open cover is an interval-valued intuitionistic fuzzy regular-open cover. □

Theorem 4.6. *The image of an* $I^v_{nearly_c}$ *space under a mapping that is both interval-valued intuitionistic fuzzy regular-almost continuous and an interval-valued intuitionistic fuzzy regular-almost open surjection is an* $I^v_{nearly_c}$ *space.*

Proof. The proof of this theorem follows a similar pattern as that of Theorem 4.2. □

Theorem 4.7. *The image of an* $I^v_{nearly_c}$ *space under an interval-valued intuitionistic fuzzy regular-strong continuity is an* I^v_c *space.*

Proof. The proof of this theorem follows a similar pattern as that of Theorem 4.2. □

Definition 4.3. *An interval-valued intuitionistic fuzzy topological space* (X,δ) *is an interval-valued intuitionistic fuzzy* S_1*-regular space iff for each λ ∈ interval-valued intuitionistic fuzzy set X can be written as* $\lambda = \vee\{\mu\in IVIFS(X) : C(\mu)\leq\lambda\}$.

Theorem 4.8. *An* $I^v_{almost_c}$ *and IVIF-*S_1*-regular space* (X,δ) *is* I^v_c.

Proof. Suppose that (X,δ) is $I^v_{almost_c}$. Let $(\lambda_i)_{i\in\Gamma}$ be an I^v_o of (X,δ). Then $\vee(\lambda_i)_{i\in\Gamma} = \tilde{1}$. From the IVIF-$S_1$-regularity of (X,δ), it follows

$$\lambda_i = \vee\{\mu_i\in IVIFS(X) : C(\mu_i)\leq\lambda_i\}.$$

Then

$$\vee\lambda_i : i\in\Gamma = \vee\mu_i : i\in\Gamma = \tilde{1}.$$

By $I^v_{almost_c}$, there exists a finite subcover $C(\mu_i) = \lambda_j \leq \lambda_i (j\leq i), i, j\in\Gamma$ such that $\vee C(\lambda_j) = \tilde{1}$. But $C(\mu_i)\leq\lambda_i$, hence $\vee(\lambda_j)\leq\vee(C\lambda_j) = \tilde{1}$, that is, $\vee\lambda_j = \tilde{1}$. Therefore (X,δ) is $I^v_{almost_c}$. □

Theorem 4.9. *An* $I^v_{nearly_c}$ *IVIF-*S_1*-regular space* (X,δ) *is* $I^v_{almost_c}$.

Proof. Suppose that (X,δ) is $I^v_{nearly_c}$. Let $(\lambda_i)_{i\in\Gamma}$ be an I^v_o of (X,δ). Then $\vee(\lambda_i)_{i\in\Gamma} = \tilde{1}$. From the IVIF-$S_1$-regularity of (X,δ), it follows that $\lambda_i = \vee\{\mu_i\in IVIFS(X) : C(\mu_i)\leq\lambda_i\}$. Then $\vee\lambda_i : i\in\Gamma = \vee\mu_i : i\in\Gamma = \tilde{1}$. By $I^v_{nearly_c}$, there exists a finite subcover $C(\mu_i) = \lambda_j \leq \lambda_i (j\leq i), i, j\in\Gamma$ such that $\vee_{j=(1,\ldots,n)} IC\lambda_j = \tilde{1}$. But $I(C\lambda_j)\leq C(\lambda_j)\leq\lambda_j = \tilde{1}$, hence $\vee_{j=(1,\ldots,n)} IC\lambda_j = \tilde{1}$. Therefore, (X,δ) is $I^v_{almost_c}$. □

5. CONCLUSIONS

The paper introduces the concepts of interval-valued intuitionistic almost (near) compactness and S_1-regular spaces and proves that if an intuitionistic fuzzy topological space (X, δ) is an S_1-regular space and interval-valued intuitionistic almost (near) compact, then it is interval-valued intuitionistic compact.

The following problems are considered in detail:

(i) Intuitionistic fuzzy retracts, which are generalizations using intuitionistic fuzzy continuity are defined and compared, and respective examples are provided.

(ii) Structural properties of interval-valued intuitionistic almost (near) compactness and S_1-regular spaces are discussed via intuitionistic fuzzy topology.

(iii) Interval-valued intuitionistic almost (near) compactness is compared with other important classes of interval-valued intuitionistic fuzzy sets, which provides a way to study compactness in a more generalized form in the future.

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Intuitsionistlikud hägusad γ-retraktid ja vahemikväärtustega intuitsionistlik peaaegu kompaktsus

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On tutvustatud intuitsionistliku hägusa γ-retrakti ja intuitsionistliku hägusa R-retrakti mõistet. On uuritud mõningaid nende uute mõistete vahelisi seoseid, toodud nende mõistete kohta näiteid ja leitud nende mõistete mõningaid omadusi. Samuti on uuritud vahemikväärtustega intuitsionistlikku peaaegu kompaktsust ja defineeritud S_1-regulaarsed ruumid. On tõestatud, et kui intuitsionistlikult hägus topoloogiline ruum on S_1-regulaarne ruum ja vahemikväärtustega intuitsionistlikult peaaegu kompaktne, siis on see ka vahemikväärtustega intuitsionistlikult kompaktne.