

Proceedings of the Estonian Academy of Sciences, 2016, **65**, 1, 68–77 doi: 10.3176/proc.2016.1.07 Available online at www.eap.ee/proceedings

CHEMISTRY

Modelling equilibrium distribution of carbonaceous ions and molecules in a heterogeneous system of CaCO₃-water-gas

Toomas Tenno^{*}, Ergo Rikmann, Ivar Zekker, Taavo Tenno, Laura Daija, and Alexey Mashirin

Institute of Chemistry, University of Tartu, Ravila 14a, 50411 Tartu, Estonia

Received 23 April 2015, accepted 26 June 2015, available online 8 February 2016

Abstract. Equilibrium processes involving dissolved CO_2 play a vital role in many biological and technological systems. Dissolved CO_2 is an available carbon source for autotrophic organisms that have great natural and technological relevancy. In many places, including northern Estonia, the soil bedrock is limestone $(CaCO_3)$, which has a relatively small ion product and hence low solubility. However, in the equilibrium system $CaCO_3 \leftrightarrow CO_3^{-2} \leftrightarrow HCO_3^{-} \leftrightarrow H_2CO_3 \leftrightarrow CO_2$ the solubility of $CaCO_3$ is elevated, which causes a relatively high alkalinity (pH > 8) of surface and ground water as a result of increased concentration of CO_3^{-2} , HCO_3^{-} , and OH^{-} ions. In this paper the equilibrium processes involving $CaCO_3$ (limestone) in liquid–gas–solid phase systems are described. The equilibrium distribution of ions and molecules in the system is described on the basis of a structural scheme. A mathematical model is presented for the calculation of pH and concentrations of ions and molecules in the equilibrium system CO_3^{-2} , HCO_3^{-} , CO_2 , Ca^{2+} , H^+ , and OH^- at different concentrations of CO_2 in its liquid phase using an iteration method. The developed model was experimentally validated.

Key words: calcium carbonate, carbon dioxide, water, equilibrium, ternary system.

INTRODUCTION

Carbon dioxide is an important component of Earth's atmosphere, constituting approximately 390–400 ppm [1,2]. Carbon dioxide is produced during many different oxidation and decomposition processes in both natural and engineered environments [3,4]. Either in free (CO₂) or bonded forms (HCO₃⁻ and CO₃²⁻), carbon dioxide is an essential component of the biosphere, lithosphere, atmosphere, and hydrosphere, with a continuous dynamic exchange of CO₂ taking place between these spheres [3,4]. For autotrophic organisms, inorganic carbon (CO₂) is the main carbon source for growth [5,6]. Carbon dioxide is used in photosynthesis by algae and plants. Microorganisms fixing CO₂ from archaeal and bacterial domains have a substantial role in a variety of biogeochemical cycles (e.g. the nitrogen

and carbon cycles) and ecosystems [7,8]. Technological applications of microbial processes based on autotrophic bacteria, for example autotrophic nitrogen removal that uses both aerobic and anaerobic ammonium-oxidizing bacteria, the so-called anammox bacteria, are growing rapidly [5,6,9]. Applications for photoautotrophic micro-algae are also being developed [10,11].

Photosynthesis of algae in natural water bodies leads to an increase in pH [12]. When CO_2 (more precisely, $(CO_2)_W$) is removed from water by photosynthesis, some HCO_3^- is converted into $(CO_2)_W$ over intermediate H_2CO_3 in order to maintain equilibrium. Consequently, some CO_3^{2-} will take up protons originating from dissociation of water and produce HCO_3^- in order to compensate for the decrease in the concentration of HCO_3^- . These two processes combined result in a new equilibrium. However, due to the release of surplus OH⁻ ions the pH value of the water increases [12]. Inorganic carbon is taken up by microbial cells in the form of

^{*} Corresponding author, Toomas.Tenno@ut.ee

dissolved CO₂, which permeates biological membranes by diffusion and results in a surplus of H⁺ ions inside cells, causing a drop in intracellular pH [13]. The growth of autotrophic cells therefore depends on the availability of dissolved CO₂. If however too much dissolved CO₂ enters the cytoplasm, causing acidification, the cells may be unable to expel the excess protons, resulting in the inhibition of cells' metabolism [14]. Additionally, this inhibitory effect of CO₂ may be linked to changes in the permeability and structure of cell membranes and enzymatic synthesis in the cytoplasm [15]. Relatively little attention appears to have been paid to studies concerning the CO₂ limitations and C/N ratio on autotrophic bacteria [9].

Limestone, which forms the soil bedrock in northern Estonia, causes relatively high alkalinity (pH > 8) of ground and surface water [16]. Equilibrium processes between carbonaceous ions and molecules dissolved in water from this solid phase (i.e. limestone bedrock, which mainly consists of CaCO₃) and the atmosphere (CO₂ in the gaseous phase) also affect treatment processes of potable water and wastewater [3,8]. This equilibrium system involves the ions and molecules CO_3^{2-} , HCO_3^{-} , CO_2 , Ca^{2+} , H⁺, OH⁻, and H₂CO₃.

Although CO_2 balance in aqueous mediums has been a subject of intensive kinetic and computational studies, a detailed mechanistic interpretation of CO_2 in water environments has yet to be established [17]. For practical purposes, an accurate mathematical model enabling the calculation of the concentrations of all dissolved ions and molecules is required.

In this paper the equilibrium system of an aqueous solution of CaCO₃ and the corresponding equilibrium

content of CO_2 in the gas and water phases is described in a structural scheme. A mathematical model applying the solution to an iterative series of equations for the calculation of the components in this three-phase system is presented. Our model also contains interfacial mass transfer in the formation of the equilibrium distribution of the components of the system.

COMPONENTS OF THEORETICAL MODEL FOR THE EQUILIBRIUM SYSTEM OF SOLID CaCO₃-WATER-CO₂ IN THE GAS PHASE

The interfacial transfer of CO_3^{2-} between solid and liquid phases and CO_2 between gas and liquid phases occurs in the ways shown in Fig. 1.

The quantity of carbonaceous ions and molecules in the liquid phase is defined as the sum of the quantity of CO_2 transferred from the gas to the liquid phase and the amount of CO_3^{2-} transferred from the solid to the liquid phase. This transition is described by a mass transfer of matter:

$$m_{\Sigma}(\mathrm{CO}_X)_{\mathrm{W}} = \Delta m(\mathrm{CO}_2) + \Delta m(\mathrm{CO}_3^{2-}).$$
(1)

After reaching equilibrium, the total amount of carbonaceous ions and molecules $m_{\Sigma}(\text{CO}_X)_W$ in the liquid phase with the volume V_W is quantitatively distributed in accordance with the equilibrium constants based of the structural scheme shown in Fig. 2.

The structural scheme suggests that H^+ ions play a central role in the establishment of equilibrium in the system and form a link between all equilibrium



Fig. 1. Formation of an equilibrium state of ions and molecules in the ternary heterogeneous system CaCO₃-water-gas. $P(CO_2)_G$ denotes the partial pressure of CO₂ in the gaseous phase; V_W is the volume of the liquid phase; $(CO_2)_W$ is the dissolved CO₂ in the liquid phase; H_2CO_3 , HCO_3^- , and CO_3^{2-} are the carbonaceous ions and molecules in the liquid phase; Ca^{2+} and CO_3^{2-} are the ions in the liquid phase and dissociation products of solid CaCO₃; $\Delta m(CO_2)$ is the amount of CO₂ transferred between the gas and liquid phases; $\Delta m(CO_3^{2-})$ is the amount of CO_3^{2-} ions transferred between the solid and liquid phases; $m_{\Sigma}(CO_X)_W$ is the total amount of carbonaceous ions and molecules in the liquid phase.

Fig. 2. Structural scheme of the equilibrium distribution of ions and molecules in the ternary heterogeneous system CaCO₃-water-gas. $k_{\rm H}$ is Henry's constant, i.e. the distribution ratio of CO₂ between water and the gaseous phase; $K_{\rm h}$ is the coefficient of the equilibrium reaction of (CO₂)_W + H₂O \leftrightarrow H₂CO₃; $K'_{\rm a1}$ and $K_{\rm a2}$ are the acid dissociation constants of H₂CO₃ and HCO₃⁻, respectively; $K_{\rm SP}$ is the solubility product of CaCO₃; $K_{\rm W}$ is the ion-product constant of water.

processes described in this theoretical model. In the model the dissociation of H_2CO_3 (the reaction product of CO_2 and H_2O) produces H^+ and HCO_3^- ions. The dissolution of CaCO₃ produces CO_3^{2-} and Ca^{2+} ions. The CO_3^{2-} ions take up H^+ ions and form HCO_3^- ions, leading to a decrease in the concentration of protons. These two processes produce a new equilibrium. Due to the release of OH^- ions by the dissociation of water molecules, pH rises. As the ion-product constant of water K_W must remain constant, the excess H^+ and OH^- ions combine, forming water molecules. The final pH of the equilibrium system depends on the concentration of CO_2 in the water phase.

Gaseous carbon dioxide dissolves in water with only a small fraction converted into carbonic acid (H₂CO₃), which is in complex equilibrium with CO₂ and its dissociated forms HCO₃⁻ and CO₃²⁻. Based on the literature, the concentration of H₂CO₃ would form less than 0.3% [8,18–20] of the equilibrium concentration of (CO₂)_W in the aqueous phase. Therefore, the concentration of the unionized species H₂CO₃ and (CO₂)_W is often summed into a single hypothetical species (CO₂^{*}), which effectively represents the concentration of all dissolved CO₂. Thus, the first acid ionization constant K_{a1} is expressed as a product of both the constants K_{h} and K'_{a1} , and the concentration of H₂CO₃ may be omitted from the calculations [8]. The constant K'_{a1} has a value of $10^{-3.5}$, thus H₂CO₃ is a fairly strong acid. However, as the product of K_h and K'_{a1} , K_{a1} has a much smaller value ($K_{a1} = 4.45 \times 10^{-7}$) [8]. Therefore, carbonic acid is usually considered a weak acid, although K_{a1} actually characterizes the acidity of the hypothetical species CO₂* [20].

A characteristic feature of the heterogeneous ternary system $CaCO_3$ -water-gas is that hydrogen is not driven out from the liquid phase to the gas phase in any form, unlike in the aqueous solutions of CaS, where hydrogen is transferred to the gas phase in the form H_2S [21].

The total amount of carbonaceous ions and molecules in the liquid phase with the volume V_W is determined by the sum of their concentrations:

$$m_{\Sigma}(\text{CO}_X)_{W} = V_{W}([\text{CO}_3^{2-}] + [\text{HCO}_3^{-}] + [\text{CO}_2]_{W}).$$
 (2)

The amount of CO_3^{2-} ions transferred from the solid phase to the liquid phase with the volume V_W equals the amount of Ca^{2+} ions simultaneously transferred from the solid phase of CaCO₃ to the water phase:

$$\Delta m(\text{CO}_3^{2-}) = V_{\rm W}[\text{Ca}^{2+}]. \tag{3}$$

The amount of CO_2 molecules transferred from the gas phase to the liquid phase is determined by the specific amount ($\Delta[CO_2]_W$) of CO_2 molecules transferred from the gas phase per unit volume (V_W) of the water phase:

$$\Delta m(\mathrm{CO}_2) = V_{\mathrm{W}} \Delta [\mathrm{CO}_2]_{\mathrm{W}}.$$
 (4)

By combining Eq. (1) with Eqs (2), (3), and (4), an equation expressing the molar balance in the water phase of the heterogeneous system is formed:

$$[CO_3^{2^-}] + [HCO_3^{-}] + [CO_2]_W = \Delta [CO_2]_W + [Ca^{2^+}].$$
(5)

The physical essence of Eq. (5) is that a difference between the total quantity of carbonaceous ions and molecules and the amount of calcium in the liquid phase occurs due to a transfer of CO_2 from the gas phase to the liquid phase. The term $\Delta[CO_2]_W$ on the right hand side of Eq. (5) contains the dimension of concentration. If the system is at equilibrium, the concentrations of all species in the liquid phase can be calculated on the basis of the following equations.

Henry's constant:

$$k_{\rm H} = \frac{[\rm CO_2]_W}{P(\rm CO_2)_G} = 3.39 \times 10^{-2}, \ \frac{\rm mol}{\rm L \times atm}; \ [18]$$
 (6)



the first acid ionization constant (the product of K_h and K'_{al}):

$$K_{a1} = \frac{[\mathrm{H}^+] \times [\mathrm{HCO}_3^-]}{[\mathrm{CO}_2]_{\mathrm{W}}} = 4.30 \times 10^{-7}, \, \mathrm{mol/L}; \, [22] \quad (7)$$

acid dissociation constant of HCO₃⁻:

$$K_{a2} = \frac{[\text{H}^+] \times [\text{CO}_3^{2^-}]}{[\text{HCO}_3^-]} = 4.70 \times 10^{-11}, \text{ mol/L; [22]}$$
(8)

solubility product of CaCO₃:

$$K_{\rm SP} = [{\rm Ca}^{2+}] \times [{\rm CO}_3^{2-}] \cong 2.80 \times 10^{-9}, \ ({\rm mol/L})^2; \ [23]$$
(9)

ion-product constant of water:

$$K_{\rm W} = [{\rm H}^+] \times [{\rm OH}^-] = 1.01 \times 10^{-14}, \ ({\rm mol/L})^2. \ [23] \ (10)$$

In the heterogeneous system CaCO₃-water-gas, charge balance can be expressed as follows:

$$2[Ca^{2+}] + [H^+] = 2[CO_3^{2-}] + [HCO_3^{-}] + [OH^{-}].$$
(11)

BASIC EQUATIONS FOR THE CHARACTERIZATION OF THE EQUILIBRIUM OF THE SYSTEM

In this paper the chemical characterization of the equilibrium distribution of carbonaceous ions and molecules in the liquid phase, as well as the dissociation products of $CaCO_3$ and water, is given as functions of the concentration of CO_2 in water:

$$([Ca^{2+}], [CO_3^{2-}], [HCO_3^{-}], [H^+], [OH^-]) = f[CO_2]_W.$$

(12)

These functions are defined by the five implicit equations given above (7-11) containing five variables of the concentrations of Ca²⁺, CO₃²⁻, HCO₃⁻, H⁺, and OH⁻, i.e. the necessary and sufficient number of equations to solve the system. The equilibrium concentration of dissolved CO₂ in the liquid phase at a given partial pressure of CO₂ in the gaseous phase is calculated using Eq. (6). The concentration of CO₂ molecules in the water phase at a defined partial pressure in the gas phase is a constant taken into consideration in the equations.

In order to calculate the concentrations of the ions Ca^{2+} , CO_3^{2-} , HCO_3^{-} , H^+ , and OH^- at a predetermined value of the concentration of $[CO_2]_W$, it is necessary to solve a series of equations (7–11) in relation to one of

the concentrations of this as a function of the $[CO_2]_W$ concentration in the liquid phase. In this function the selected parameter accounts for all the remaining parameters of the equilibrium system. The complete solution to the system of equations in relation to the concentration of H^+ ions is based on the following fourth-degree equation:

$$2\frac{K_{\rm SP}[{\rm H}^+]^2}{K_{a1} \times K_{a2}[{\rm CO}_2]_{\rm W}} + [{\rm H}^+]$$

= $2K_{a1} \times K_{a2} \frac{[{\rm CO}_2]_{\rm W}}{[{\rm H}^+]^2} + K_{a1} \frac{[{\rm CO}_2]_{\rm W}}{[{\rm H}^+]} + \frac{K_{\rm W}}{[{\rm H}^+]}.$
(13)

The analytic solution of the given fourth-degree Eq. (13) is complicated. Therefore, in practice simplified solutions for specific individual cases under real conditions are used. For most applications in the pH range 6–9, determining the approximate concentration of $[H^+]_{apx}$ considering equilibrium distribution as a partial solution is proposed, simplifying Eq. (13) to Eq. (14):

$$[\mathrm{H}^{+}]_{\mathrm{apx}} \cong \sqrt[3]{\frac{(K_{\mathrm{a1}}[\mathrm{CO}_{2}]_{\mathrm{W}})^{2}K_{\mathrm{a2}}}{2K_{\mathrm{SP}}}}.$$
 [22] (14)

Our aim was to determine the parameters of the equilibrium distribution of ions and molecules in the liquid phase of the ternary heterogeneous system CO_2 - $HCO_3^--CO_3^{2-}$ for a wider range of pH, i.e. from 7 to 12. Therefore, we converted Eq. (13) to a form suitable for iterative Eq. (15). Using Eq. (15), accurate results should also be obtained for any values of pH within the range 9–12:

 $[H^+] =$

$$\sqrt[3]{\frac{K_{a1} \times K_{a2} [CO_2]_W}{2K_{SP}}} \left\{ K_{a1} [CO_2]_W \left(2\frac{K_{a2}}{[H^+]} + 1 \right) + K_W - [H^+]^2 \right\}.$$
(15)

The calculation of the equilibrium concentrations of Ca²⁺, CO₃²⁻, and HCO₃⁻ and pH was performed on the basis of the value for the equilibrium concentration of H⁺ ions obtained by an iterative method based on Eq. (15) using different values for constant concentrations of CO₂ in the range of $[CO_2]_W \cong (2.74 \times 10^{-11} \div 8.02 \times 10^{-1}) \text{ mmol/L}$ with respective equilibrium values of $P(CO_2)_G \cong (8.08 \times 10^{-7} \div 2.37 \times 10^4)$ ppm. Results of these calculations are presented in Table 1.

The data in Table 1 show that in the given range of concentrations of $[CO_2]_W \cong 2.74 \times 10^{-11} \text{ mmol/L}$ to $[CO_2]_W \cong 8.02 \times 10^{-1} \text{ mmol/L}$, the concentration of HCO_3^- and pH have a monotonous dependence

P(CO ₂) _G , ppm	[CO ₂] _W , mmol/L	[HCO ₃ ⁻], mmol/L	[CO ₃ ^{2–}] mmol/L	$\Sigma[CO_X]_W,$ mmol/L	[Ca ²⁺], mmol/L	pН
$\begin{array}{c} 8.08 \times 10^{-7} \\ 2.55 \times 10^{-5} \\ 2.95 \times 10^{-4} \\ 2.95 \times 10^{-3} \\ 2.95 \times 10^{-2} \\ 2.95 \times 10^{-1} \end{array}$	$\begin{array}{c} 2.74 \times 10^{-11} \\ 8.66 \times 10^{-10} \\ 1.00 \times 10^{-8} \\ 1.00 \times 10^{-7} \\ 1.00 \times 10^{-6} \\ 1.00 \times 10^{-5} \end{array}$	$\begin{array}{c} 1.18\times 10^{-5}\\ 1.18\times 10^{-4}\\ 6.01\times 10^{-4}\\ 2.77\times 10^{-3}\\ 1.24\times 10^{-2}\\ 4.88\times 10^{-2} \end{array}$	$\begin{array}{c} 5.54\times 10^{-3}\\ 1.75\times 10^{-3}\\ 3.94\times 10^{-3}\\ 8.37\times 10^{-3}\\ 1.67\times 10^{-2}\\ 2.60\times 10^{-2} \end{array}$	$\begin{array}{c} 5.66 \times 10^{-4} \\ 1.87 \times 10^{-3} \\ 4.55 \times 10^{-3} \\ 1.11 \times 10^{-2} \\ 2.90 \times 10^{-2} \\ 7.48 \times 10^{-2} \end{array}$	5.05 1.60 0.71 0.33 0.17 0.11	12.00 11.50 11.15 10.81 10.46 10.06
6.92×10^{-1}	2.35×10^{-5}	7.63×10^{-2}	2.71×10^{-2}	1.03×10^{-1}	0.10	9.88
$\begin{array}{c} 2.95 \times 10^{0} \\ 2.95 \times 10^{1} \\ 8.85 \times 10^{1} \\ 2.95 \times 10^{2} \end{array}$	$\begin{array}{c} 1.00\times 10^{-4}\\ 1.00\times 10^{-3}\\ 3.00\times 10^{-3}\\ 1.00\times 10^{-2} \end{array}$	$\begin{array}{c} 1.49\times 10^{-1}\\ 3.60\times 10^{-1}\\ 5.28\times 10^{-1}\\ 7.95\times 10^{-1}\end{array}$	$\begin{array}{c} 2.42\times 10^{-2} \\ 1.41\times 10^{-2} \\ 1.01\times 10^{-2} \\ 6.91\times 10^{-3} \end{array}$	$\begin{array}{c} 1.73\times 10^{-1}\\ 3.75\times 10^{-1}\\ 5.41\times 10^{-1}\\ 8.12\times 10^{-1}\end{array}$	0.12 0.20 0.28 0.41	9.54 8.92 8.61 8.27
$\begin{array}{l} 3.90 \times 10^2 \\ 8.85 \times 10^2 \\ 2.95 \times 10^3 \\ 2.37 \times 10^4 \end{array}$	$\begin{array}{c} 1.32\times 10^{-2}\\ 3.00\times 10^{-2}\\ 1.00\times 10^{-1}\\ 8.02\times 10^{-1} \end{array}$	8.74×10^{-1} 1.15 1.72 3.45	$\begin{array}{c} 6.31\times 10^{-3}\\ 4.82\times 10^{-3}\\ 3.24\times 10^{-3}\\ 1.62\times 10^{-3}\end{array}$	8.93×10^{-1} 1.19 1.82 4.25	0.44 0.58 0.86 1.73	8.19 7.95 7.60 7.00

Table 1. Equilibrium concentrations of $[CO_2]_W$, HCO_3^- , CO_3^{2-} , and Ca^{2+} and pH in the system of gas-water-solid CaCO₃ at t = 25 °C. Boldface designates extremal of Ca^{2+} and CO_3^{2-} at $[CO_2]_W \cong 2.35 \times 10^{-5} \text{ mmol/L}$

on the concentration of $[CO_2]_W$. The data were characterized by a minimum value for solubility of CaCO₃ $[Ca^{2+}]_{min} \cong 10^{-1}$ mmol/L and a maximum value of $[CO_3]_{max} \cong 2.71 \times 10^{-2}$ mmol/L at the value of $[CO_2]_W \cong 2.35 \times 10^{-5} \text{ mmol/L}$. In the range of $[CO_2]_W \cong (2.35 \times 10^{-5} \div 2.74 \times 10^{-11})$ mmol/L the corresponding value for the solubility of CaCO₃ increases according to the equation for the solubility product of CaCO₃ (K_{SP}) due to a decrease in the concentration of CO_3^{2-} from $\cong 2.71 \times 10^{-2}$ mmol/L to $\cong 5.54 \times 10^{-3}$ mmol/L. The decrease in the concentration of CO_3^{2-} ions calculated using this model occurs due to the decrease in the concentration of all carbonaceous species in the water phase. A graphical expression of the dependence of pH and the solubility of CaCO₃ on the content of $[CO_2]_W$ in the liquid phase is presented in Fig. 3. A logarithmic scale was used for the values of $[CO_2]_W$ concentration.

The concentration of CO_2 in the ambient air also depends on the season, with an annual fluctuation of approximately 3–9 ppm [2]. The content of CO_2 in the ambient air is normally the highest in March–April and the lowest in September–October, roughly following the Northern Hemisphere's growing season. There is a dynamic balance between the atmosphere and hydrosphere, which is affected by oscillations in meteorological conditions such as atmospheric pressure and temperature [2]. Where $P(CO_2)_G$ levels differ from normal atmospheric values, the solubility of CaCO₃ is correspondingly affected. This situation arises when biological respiration, aerobic or anaerobic, releases large amounts of CO_2 , as for example in soil containing a substantial population of microorganisms or in water bodies with active algal growth.

The graph in Fig. 3 also shows the dependence of pH and the solubility of $CaCO_3$ on the content of CO_2 in the water phase $(pH_{apx} \text{ and } [Ca^{2+}]_{apx})$, as calculated by the approximate method using Eq. (13). Simplification of Eq. (13) to give Eq. (14), with the goal of the elimination of some terms, is justified on the grounds that such elimination does not create a significant error [24]. The solubility of CaCO₃ calculated using Eq. (13) produces a significant error if pH > 9 (Fig. 3).

The approximate method has been mentioned in many sources [3,4,8,24]. In conformity with Eq. (13) the solubility of Ca²⁺ monotonously decreases along with decreasing CO₂ concentrations (as shown with a dashed line in Fig. 3) [4]. This is in contrast with our model, demonstrating that the solubility of Ca²⁺ passes through a minimum at $[CO_2]_W \cong 2.35 \times 10^{-5}$ mmol/L and increases again with decreasing $[CO_2]_W$.

Figure 3 shows the graph has a well-defined quasilinear region dependence of $pH = f(\log[CO_2]_W)$ at pH < 9. This dependence is described by a linear equation following the approximate expression [22]:

$$pH_{apx} \cong \frac{1}{3} \{ \log(2K_{SP}) - \log K_{a2} - 2(\log K_{a1} + \log[CO_2]_W) \}.$$
(16)



Fig. 3. Dependence of pH and $[Ca^{2+}]$ on $[CO_2]_W$. *A* refers to the content of $[CO_2]_W$ in the liquid phase at $P(CO_2)_G \cong 3.9 \times 10^2$ ppm [1] in the ambient air at standard conditions (25 °C, 100 kPa).

INTERFACIAL TRANSPORT OF SUBSTANCES IN A HETEROGENEOUS SYSTEM AT THE EQUILIBRIUM DISTRIBUTION OF CaCO₃-CO₃²⁻-HCO₃⁻-(CO₂)_W-(CO₂)_G

During the formation of an equilibrium state, all transferred calcium ions remain in the liquid phase. Therefore, the transfer of CO_3^{2-} from the solid phase to the liquid phase is determined by the amount of Ca^{2+} ions in the liquid phase:

$$\Delta [\mathrm{CO}_3^{2-}]_{\mathrm{W}} = [\mathrm{Ca}^{2+}], \qquad (17)$$

where $\Delta[CO_3^{2^-}]_W$ is the amount of $CO_3^{2^-}$ transferred from the solid (CaCO₃) to the liquid phase per unit volume of the liquid.

The amount of CO_2 transferred from the gas phase to the liquid phase per unit volume of the liquid during the formation of equilibrium in the system containing carbonaceous ions and molecules can be derived from Eqs (1) and (17) as:

$$\Delta[\operatorname{CO}_2]_{\mathrm{W}} = \Sigma[\operatorname{CO}_X]_{\mathrm{W}} - [\operatorname{Ca}^{2+}], \qquad (18)$$

where $\Delta[CO_2]_W$ is the amount of CO_2 transferred to the liquid phase per unit volume of the liquid and $\Sigma[CO_X]_W$ is the total amount of carbonaceous ions and molecules in the liquid phase at equilibrium.

The data in Table 1 and in Fig. 4 show that at a value of $[CO_2]_W \cong 2.35 \times 10^{-5}$ mmol/L, the total equilibrium concentration of carbonaceous ions and molecules in the liquid phase is nearly equal to the concentration of calcium ions:

$$\Sigma[\operatorname{CO}_X]_{\mathrm{W}} \cong [\operatorname{Ca}^{2+}], \tag{19}$$

$$\Delta[\operatorname{CO}_2]_{\mathrm{W}} = \Sigma[\operatorname{CO}_X]_{\mathrm{W}} - [\operatorname{Ca}^{2^+}] \cong 0.$$
 (20)

Consequently, there is no transfer of CO₂ from the gas phase, which corresponds to the case of equilibrium in a closed system without the gas phase. At the value $[CO_2]_W = 1.32 \times 10^{-2} \text{ mmol/L}$ the total equilibrium



Fig. 4. Functions of the phase transfer of $\Delta [CO_3^{2-}] = f[CO_2]_W$ and $\Delta [CO_2]_W = f[CO_2]_W$.

concentration of calcium ions was approximately less than twice the sum of carbonaceous ions and molecules:

$$\Sigma[\mathrm{CO}_X]_{\mathrm{W}} \approx 2[\mathrm{Ca}^{2+}]. \tag{21}$$

Thus, the transfer of CO_2 from the gas phase accounts for nearly half the amount of carbonaceous ions and molecules in the liquid phase, with the other half of the carbonaceous ions and molecules resulting from the transfer of CO_3^{2-} from the solid phase to the water phase (see Fig. 4).

The dependence of the phase transfer of carbonaceous ions and molecules on the concentration of dissolved carbon dioxide $\Delta[CO_3^{2-}] = f[CO_2]_W$ and $\Delta[CO_2]_W = f[CO_2]_W$ is shown in Fig. 4.

The region $[CO_2]_W > 2.35 \times 10^{-5} \text{ mmol/L}$ was characterized by the condition $\Delta [CO_2]_W > 0$, i.e. transfer of CO₂ from the gas phase to the liquid phase, while the region $[CO_2]_W < 2.35 \times 10^{-5} \text{ mmol/L}$ was characterized by the condition $\Delta [CO_2]_W < 0$, i.e. transfer of CO₂ from the liquid phase to the gas phase.

The value $[CO_2] \cong 2.35 \times 10^{-5}$ mmol is characterized by the condition $\Delta [CO_2]_W \cong 0$, i.e. lack of a transfer of CO₂ between the liquid and the gaseous phase, which corresponded to the case of equilibrium in a closed system.

EXPERIMENTAL VALIDATION OF THE THEORETICAL MODEL

A series of experiments were performed in order to validate the theoretical model CaCO₃-water-gas. A five-necked Simax 1 L glass bottle was used as a reaction cell and thermostatted to 25.0±0.1 °C in a water bath (ELMI SkyLine). The reaction cell was equipped with a pH glass electrode (Mettler Toledo InLab[®] Routine, pH = 0-14), temperature sensors (Jenway model 027500), and two gas inlets enabling the channelling of the gas flow through the liquid phase using a disperger and the gas phase without disperging the gas through the liquid phase. The gas outlet was equipped with a water lock in order to prevent penetration of ambient air into the reaction medium. The pH and temperature values were measured and recorded using a pH meter (Jenway type 3520) and Dataway software.

For the liquid phase, MilliQ water was used with added KCl (100 mg/800 mL) to ensure sufficient conductivity for accurate pH measurements at low concentrations of substrate. The gas mixtures used for purging the liquid phase are listed in Table 2. The gas mixtures and Argon 5.0 and 6.0 (argon containing <10 ppm or <1 ppm impurities, respectively) were purchased from AGA Eesti Ltd. For aeration the liquid phase was purged with ambient air until the pH value dropped below 6 in the liquid phase; then solid CaCO₃ (CAS # 471-34-1, 99.95%, obtained from Sigma-Aldrich) was added to the liquid phase with discontinued

aeration. Experiments of purging the liquid phase with argon containing small concentrations of CO_2 were also carried out (Table 2). In these experiments, purging was continued until a stable pH value was reached and then solid CaCO₃ was added. During the experiment the liquid phase was agitated at 200 rpm using a magnetic stirrer. At the end of the experiment, stirring was discontinued and solids were let to settle for 30 min.

From settled samples, total hardness and alkalinity were determined titrimetically. In addition, from the filtrated samples (0.45 μ m membrane filter), the concen-

Table 2. Experimentally measured and model-predicted values for the concentration of Ca²⁺

Composition of gas atmosphere	[Ca ²⁺] measured, mmol/L	[Ca ²⁺] theoretically predicted, mmol/L	<i>p</i> -value	pH value measured	pH value theoretically predicted	<i>p</i> -value
A. Ambient air, 390 ppm CO_2	0.421 ± 0.005	0.444	0.24	8.09 ± 0.03	8.19	0.26
<i>B</i> . Ar 5.0, <1.7 ppm CO ₂	0.113 ± 0.012	< 0.108	0.80	9.83 ± 0.03	9.67	0.62
<i>C</i> . Ar, 0.836 ppm CO ₂	0.100 ± 0.016	0.104	0.84	10.02 ± 0.03	9.88	0.24
<i>D</i> . Ar 6.0, <0.1 ppm CO ₂	0.120 ± 0.007	< 0.126	0.52	10.06 ± 0.03	10.25	0.23
<i>E</i> . Ar, previously purified with 5 M NaOH	0.145 ± 0.009	Not calculated		10.24 ± 0.03	Not calculated	



Fig. 5. Experimentally determined values for the concentration of soluble Ca^{2+} and pH. Experimental data points *A*–*E* are specified in Table 2.

tration of Ca²⁺ was determined using ion chromatography (type Dionex ICS-1000 ion chromatograph, suppressor Dionex CERS 500, 4 mm, column type Dionex IonPac CS12A, 4 mm, with a CG12A 4 mm guard column). Total hardness was determined by complexometrical titration of Ca²⁺ with 0.001 M EDTA solution. The experimental results are shown in Table 2 and Fig. 5. In order to evaluate statistical similarity of data from the experiments and theoretical model, *p*-values of two-way *t*-test for both data sets were calculated ($\alpha = 0.05$).

Experimentally determined values for the concentration of soluble Ca^{2+} , in accordance with the model developed in this study, showed a minimum value at a $P(CO_2)_G$ of 0.836 ppm and a $[CO_2]_W$ value of 2.82×10^{-5} mmol/L, thus providing evidence in favour of the efficacy of the theoretical model developed in this study. The statistical similarity between experimental data and calculated data obtained by the theoretical model was shown by the two-way *t*-test (Table 2).

CONCLUSIONS

The developed model for equilibrium distribution in the system solid CaCO₃-water-gas containing CO₂ can be used for the determination of the concentrations of all components in the water phase over a wide range of natural and industrial conditions, based on the known content of CO_2 in the water phase in equilibrium with the gas phase. The developed model also describes interfacial mass transfer of CO₃²⁻ between the solid and liquid phases and CO₂ between the liquid and gas phases. Quantitative evaluation of the equilibrium distribution of ions and molecules in the system CaCO₃-CO₃²⁻-HCO₃⁻-CO₂ at an equilibrium state showed a value of $pH \cong 8.2$, which is characteristic for a situation where limestone (mainly consisting of CaCO₃) bedrock is in contact with water and the atmosphere. The value $pH \cong 8.2$ corresponds to the content of CO₂ in water in equilibrium with atmospheric air (gas phase) containing $P(CO_2)_G \cong 390 \text{ ppm}$ at a temperature of $t \cong 25 \,^{\circ}C$. Quantitative evaluation of the equilibrium distribution of the species $CaCO_3 - CO_3^2 - HCO_3^2 - CO_2$ in a ternary heterogeneous system identified a relevant minimum solubility of $s [CaCO_3]_{min} \cong 0.1 \text{ mmol/L}$. The model allows for assessment of the impact of anthropogenic processes on the natural environment and could be usefully applied by environmental technologists, for instance in wastewater treatment, including nitrogen removal from reject water of biomethane production. The model is useful for developing innovative methods for the measurement of $[CO_2]_W$. This will be a topic for future research by our group.

ACKNOWLEDGEMENTS

The study was supported by projects SF0180135s08, SLOTI08262, SLOKT11027T, SLOKT11119, and IUT20-16.

REFERENCES

- Ahrens, C. D. Essentials of Meteorology: An Invitation to the Atmosphere. Brooks/Cole, Belmont, CA, 2011, p. 5.
- U.S. National Oceanic & Atmospheric Administration, Earth System Research Laboratory, http://www.esrl.noaa.gov/ gmd/ccgg/trends/ (accessed 13.02.2015).
- Baird, C. and Cann, M. *Environmental Chemistry*. 3rd ed. W. H. Freeman & Co, New York, 2005.
- vanLoon, G. W. and Duffy, S. J. *Environmental Chemistry:* A Global Perspective. 2nd ed. Oxford University Press, 2007, pp. 241–244.
- Zekker, I., Rikmann, E., Loorits, L., Tenno, T., Fritze, H., Tuomivirta, T., et al. Start-up of low temperature anammox in UASB from mesophilic yeast factory anaerobic tank inoculum. *Environ. Technol.*, 2015, 36, 214–225.
- Zekker, I., Rikmann, E., Tenno, T., Kroon, K., Vabamäe, P., Salo, E., et al. Deammonification process start-up after enrichment of anammox microorganisms from reject water in a moving-bed biofilm reactor. *Environ. Technol.*, 2013, 34, 3095–3101.
- Saini, R., Kapoor, R., Kumar, R., Siddiqi, T. O., and Kumar, A. CO₂ utilizing microbes – a comprehensive review. *Biotechnol. Adv.*, 2011, 29, 949–960.
- Snoeyink, V. L. and Jenkins, D. Water Chemistry. John Wiley & Sons, Inc., New York, 1980, pp. 160, 448.
- Zekker, I., Rikmann, E., Tenno, T., Seiman, A., Loorits, L., Kroon, K., et al. Nitritating-anammox biomass tolerant to high DO concentration and C/N ratio in treatment of yeast factory wastewater. *Environ. Technol.*, 2014, 35, 1565–1576.
- Farooq, W., Suh, W. I., Park, M. S., and Yang, J.-W. Water use and its recycling in microalgae cultivation for biofuel application. *Bioresource Technol.*, 2015, 184, 73–81.
- Zhang, X. L., Yan, S., Tyagi, R. D., and Surampalli, R. Y. Biodiesel production from heterotrophic microalgae through transesterification and nanotechnology application in the production. *Renew. Sust. Energ. Rev.*, 2013, 26, 216–223.
- Uusitalo, J. Algal carbon uptake and the difference between alkalinity and high pH ("alkalization"), exemplified with a pH drift experiment. *Sci. Mar.*, 1996, 60(Supl. 1), 129–134.
- Fraiha, M., Ferraz, A. C. de O., and Biagi, J. D. Pretreatment of thermoduric spores in CO₂ modified atmosphere and their survivability during food extrusion. *Ciência e Tecnologia de Alimentos*, 2011, **31**(1), 167–171.
- Garcia-Gonzalez, L., Geeraerd, A. H., Spilimbergo, S., Elst, K., Van Ginneken, L., Debevere, J., et al. High pressure carbon dioxide inactivation of micro-

organisms in foods: the past, the present and the future. *Int. J. Food Microbiol.*, 2007, **117**, 1–28.

- Zekker, I., Rikmann, E., Tenno, T., Vabamäe, P., Kroon, K., Loorits, L., et al. Effect of HCO₃⁻ concentration on anammox nitrogen removal rate in moving bed biofilm reactor. *Environ. Technol.*, 2012, 33, 2263–2271.
- 16. Kalm, V. and Gorlach, A. Impact of bedrock surface topography on spatial distribution of Quaternary sediments and on the flow pattern of late Weichselian glaciers on the East European Craton (Russian Plain). *Geomorphology*, 2014, **207**(15), 1–9.
- Stirling, A. and Pápai, I. H₂CO₃ forms via HCO₃⁻ in water. J. Phys. Chem. B, 2010, **114**, 16854–16859.
- Stumm, W. and Morgan, J. J. Aquatic Chemistry: Chemical Equilibria and Rates in Natural Waters. 3rd ed. John Wiley & Sons, Inc., New York, 1996, p. 214.

- Chang, R. Physical Chemistry with Applications to Biological Systems. 2nd ed. Williams College, Macmillan Publishing Co., Inc., New York, 1990, p. 320.
- Moran, D. Carbon dioxide degassing in fresh and saline water. I: Degassing performance of a cascade column. *Aquat. Eng.*, 2010, 43, 29–36.
- Zekker, I., Tenno, T., Selberg, A., and Uiga, K. Dissolution modeling and experimental measurement of CaS–H₂O binary system. *Chinese J. Chem.*, 2011, 29, 2327–2336.
- Segal, B. G. Chemistry, Experiment and Theory. John Wiley & Sons, Inc., New York, 1989, pp. 363–365.
- 23. Dean, J. A. Lange's Handbook of Chemistry. McGraw-Hill, Inc., New York, 1992, pp. 7, 8.
- Skoog, D. A., West, D. M., and Holler, F. J. Fundamentals of Analytical Chemistry. 6th ed. Saunders College Publishing, 1992.

Süsinikku sisaldavate ioonide ja molekulide tasakaalulise jaotuse modelleerimine heterogeenses süsteemis CaCO₃-vesi-gaasifaas

Toomas Tenno, Ergo Rikmann, Ivar Zekker, Taavo Tenno, Laura Daija ja Alexey Mashirin

Lahustunud CO₂ osalusel kulgevad tasakaalulised protsessid on paljudes bioloogilistes ja tehnoloogilistes süsteemides väga olulised. Lahustunud CO₂ on peamine süsinikuallikas autotroofsetele organismidele, kel on suur tähtsus looduslikes ökosüsteemides ja reoveepuhastuse tehnoloogias. Piirkondadele, kus pinnase aluskivim on lubjakivi (CaCO₃), sealhulgas Põhja-Eesti, on omane pinna- ja põhjavee suhteliselt kõrge leelisus (pH > 8) ning karedus. Kuigi CaCO₃-le on iseloomulik madal lahustuvuskorrutise (K_{SP}) väärtus ja sellest tulenevalt vähene lahustuvus, toimub heterogeenses tasakaalulises süsteemis CaCO₃ \leftrightarrow CO₃²⁻ \leftrightarrow HCO₃⁻ \leftrightarrow H₂CO₃ \leftrightarrow CO₂ CaCO₃ lahustuvuse suurenemine, mis põhjustab pinna- ja põhjavees CO₃²⁻, HCO₃⁻ ning OH⁻ ioonide kontsentratsiooni tõusu.

Käesolevas artiklis on kirjeldatud CaCO₃ (lubjakivi) osalusel kulgevaid tasakaalulisi protsesse vedeliku-gaasitahkise faasis. Ioonide ja molekulide tasakaalulist jaotust süsteemis on kirjeldatud struktuurskeemi abil. On esitatud matemaatiline mudel pH ja CO₃²⁻, HCO₃⁻, CO₂, Ca²⁺, H⁺ ning OH⁻ kontsentratsioonide arvutamiseks tasakaalulises süsteemis vedelfaasi erinevate CO2 kontsentratsioonide korral, kasutades iteratsioonimeetodit. Väljatöötatud mudel on leidnud katselist kinnitust. Mudelit saab kasutada vedelfaasi kõikide koostisosade kontsentratsioonide määramiseks erinevate looduslike ja tehnoloogiliste süsteemide iseloomulikel tingimustel, põhinedes teadaolevale CO₂ kontsentratsioonile gaasifaasiga tasakaalus olevas veefaasis. Mudel kirjeldab ka CO₃²⁻ massiülekannet tahke ja vedela faasi ning CO₂ massiülekannet vedela ja gaasilise faasi vahel. Ioonide ja molekulide õhuga tasakaalulise süsteemi jaotuse kvantitatiivne hindamine heterogeenses süsteemis $CaCO_3 \leftrightarrow CO_3^{2-} \leftrightarrow HCO_3^{-} \leftrightarrow H_2CO_3 \leftrightarrow CO_2$ on iseloomustatav pH väärtusega 8,2, mis on iseloomulik olukorrale, kus lubjakivi (koosneb peamiselt CaCO₃-st) pinnase aluskivimina on kokkupuutel vee ning atmosfääri õhuga. Väärtus pH \cong 8,2 vastab CO₂ sisaldusele vees temperatuuril $t \cong 25$ °C, mis on tasakaalus atmosfääriõhuga (gaasifaasis), CO₂ sisaldusel $P(CO_2)_G \cong 390$ ppm. Ioonide ja molekulide tasakaalulise jaotuse kvantitatiivne hindamine heterogeenses süsteemis $CaCO_3 \leftrightarrow CO_3^{2-} \leftrightarrow HCO_3^{-} \leftrightarrow H_2CO_3 \leftrightarrow CO_2$ tuvastas iseloomuliku CaCO₃ lahustuvuse miinimumi s [CaCO₃]_{min} \approx 0,1 mmol/L. Mudel võimaldab hinnata inimtekkeliste protsesside mõju looduskeskkonnale ja on rakendatav keskkonnatehnoloogias, näiteks reoveepuhastuses (sealhulgas lämmastikuärastusel biometaani tootmisel tekkiva vädu töötlemisel). Mudel võimaldab arendada innovaatilisi CO₂ kontsentratsiooni mõõtmise meetodeid, mis on uurimisgrupi üks tulevaste uuringute teemasid.