Simulation of solitary wave propagation in carbon fibre reinforced polymer

Martin Lints[a,b*], Andrus Salupere[a], and Serge Dos Santos[b]

[a] CENS, Institute of Cybernetics at Tallinn University of Technology, Akadeemia tee 21, 12618 Tallinn, Estonia
[b] INSA Centre Val de Loire, Blois Campus, 3 Rue de la Chocolaterie, CS 23410, F-41034 Blois Cedex, France

Abstract. The emergence and propagation of solitary waves is investigated for carbon fibre reinforced polymer using numerical simulations for Non-Destructive Testing (NDT) purposes. The simulations are done with the Chebyshev collocation method. The simplest laminate model is used for the periodical structure of the material from which dispersion will arise. Classical and nonclassical nonlinearities are introduced in the constitutive equation. The balance of the dispersion and nonlinearity is analysed by studying the shape-changing effects of the medium on the initial input pulse and the possibility of solitary wave propagation is considered. Future applications of solitary waves for nonlinear medical imaging and NDT of materials are discussed.

Key words: CFRP, solitary waves, Non-Destructive Testing, TR–NEWS, nonlinearity, dispersion.

1. INTRODUCTION

The recent ten years have seen considerable development of optimized signal processing methods for improving nonlinear Non-Destructive Testing (NDT) methods derived from Nonlinear Elastic Wave Spectroscopy (NEWS) and supplemented by symmetry invariance and Time Reversal (TR). The emerging TR–NEWS method is a useful tool for microcracks detection of various complex samples [1], but also recently for the localization of nonlinear scatterers in a wide sense [2]. TR–NEWS signal processing is performed with symmetrization of coded excitation using cross-correlation, pulse-inversion [3], or chirp-coded schemes, which are promising alternatives to frequency coding. The response to positive and negative excitations enables to extract the nonlinear signature of the tested sample.

In materials with nonlinear and dispersive properties, solitary waves could be used for NDT [4,5]. They are stable in propagation and have elastic interactions due to the balance between the nonlinearity and dispersion. This robustness could improve the monitoring capabilities of layered, granular, or functionally graded materials. It is well known that in such a medium, dispersion and nonlinearity could be combined in a way that solitonic

propagation could be observed. The dispersion can be caused by the material microstructure [6] or layers [7,8], and the nonlinearity by the microdamage or soft inclusions [9]. Using solitonic excitation, a medium with these properties could be analysed. The solitary waves can experience a phenomenon called 'selection' where the amplitude and velocity of a solitary wave tend to finite values, which depend on the nonlinearity and dispersion [10–12]. In some microstructured models, the solitary wave propagation can also be sensitive to the ratio of macro- and microstructural dispersions and a general 'shape' of the initial profile [13], which could likewise be used for diagnostic purposes.

Carbon Fibre Reinforced Polymers (CFRPs) are being increasingly used for applications requiring both a high strength to weight ratio and reliability, for example in aerospace, automotive, and naval industries. Therefore demand for the robustness of NDT of layered composites is rising. The material is geometrically complex and has several micro-scales: firstly, the scale of individual carbon fibres that make up a single yarn; secondly, the scale of individual yarns from which the fabric is woven; and finally in 2D or 3D cases, the scale of individual layers (carbon fabric and polymer). This makes the use of conventional NDT techniques difficult, which is why

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* Corresponding author, martin.lints@cens.ioc.ee

this work analyses the potential use of solitary waves for testing a material with nonclassical nonlinearities at multiscale level. In this work the plies of the composite are regarded as homogeneous orthotropic materials. This multiscale complexity also justifies the use of methods such as TR–NEWS because they have shown extreme efficiency in complex media, such as composites and biological tissues.

The numerical simulations are done by the Chebyshev collocation method with Chebyshev polynomials used for approximating physical quantities and finding the spatial derivatives. The simulations take into account the layered character of the material and are performed for a 144 layered CFRP test sample. In addition to the dispersion arising from the layered configuration, the material is assumed to be weakly nonlinear. The goal is to determine the influence of nonlinearity on the character of the propagating waves. The main attention is paid to the formation of solitary waves. The conditions for the emergence of a solitary wave and its propagating characteristics are analysed for future use in physical experiments.

This paper is presented as follows. Firstly, the model of the CFRP material is described, the classical and nonclassical nonlinearities are introduced into the governing equations, and the key points about the numerical method are described. Secondly, the simulation results are presented and then analysed for the effect of a small but global nonclassical nonlinearity.
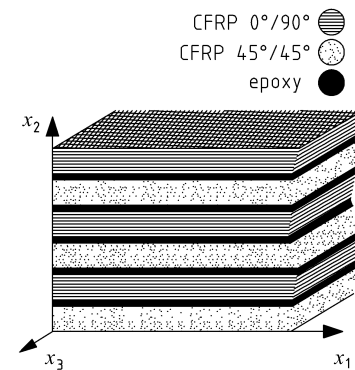
2. MODEL

The modelled material is a CFRP block with a thickness of 43 mm, consisting of 144 layers (Fig. 1). It is composed of fabric woven from yarns of fibre and impregnated with epoxy. The cross-section of the yarns is of elliptical shape (Fig. 2) and the material has inclusions of pure epoxy, so a wave propagating through the material will encounter yarns (fibres with epoxy) and areas of pure epoxy. The simplest material model for the test object is the laminate model (Fig. 3) in which (i) the
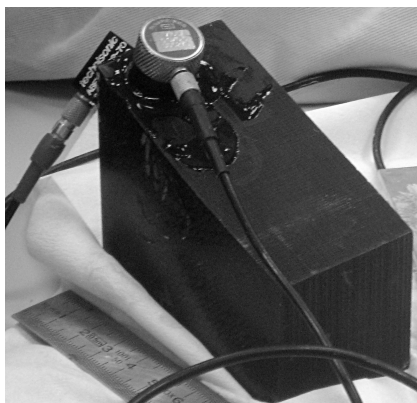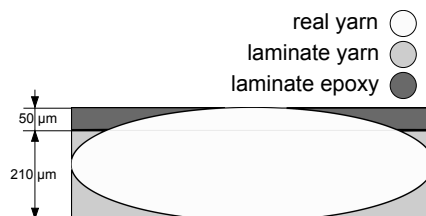

Fig. 1. Carbon Fibre Reinforced Polymer (CFRP) block of 144 layers tested with ultrasonic of NDT.
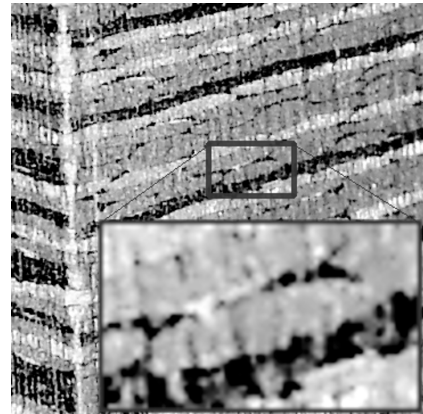
Fig. 2. Close-up image of the structure of the test object.

Fig. 3. Material model of the CFRP block.

Fig. 4. The material is modelled as a laminate, with the thickness of individual laminae proportional to the cross-sectional area of the material.

material consists of homogeneous layers, (ii) each layer has its own elasticity properties, and (iii) dispersion arises due to the periodical discontinuity of the properties. The widths of the layers are proportional to the area of the cross-section of the yarn (Fig. 4) and are here modelled as laminates of constant thickness (thickness $h_e = 50$ μm for the epoxy layer and $h_{CFRP} = 210$ μm for the pure CFRP layer). The longitudinal wave modulus for the epoxy layer $E_e = 6.5$ GPa and for the CFRP layer $E_{CFRP} = 13.6$ GPa.

2.1. Mathematical model

The deformations are assumed to be small: $\varepsilon_{kl} = \frac{1}{2}\left(u_{k,l} + u_{l,k}\right)$. The Cauchy's equations governing the

wave motion in each piecewise continuous layer are

$$\begin{cases} \sigma_{kl,k} + \rho \ddot{u}_l = 0, \\ \sigma_{kl} = \sigma_{lk}. \end{cases} \quad (1)$$

The constitutive equation is

$$\sigma = \alpha E \left(\varepsilon - \beta \varepsilon^2 \right). \quad (2)$$

In the above equations σ denotes stress, ε denotes strain, u denotes displacement, ρ is the density of the material, and E is the modulus of elasticity. The Einstein's summation convention is used. An index after a comma denotes a derivative in that direction. Weak classical nonlinearity is given by β and nonclassical nonlinearity by α. Here the nonclassical nonlinearity means that the material can have an abrupt change in the elasticity modulus (in this work on $\varepsilon = 0$). This permits strong nonlinear effects in cases of small strain. Nonclassical nonlinearity parameter α allows the material to be weaker in tension than in compression [14]: $\alpha \leq 1$ if $\varepsilon \geq 0$ and $\alpha = 1$ always if $\varepsilon < 0$. There is no nonclassical nonlinearity if $\alpha = 1$ for all ε. In order to use the pulse-inversion method [3], dynamic boundary conditions with both positive and negative polarities and with temporal extent τ were used.

$$\sigma(0,t) = \begin{cases} \pm 35 \cdot 10^3 \left(1 + \cos \left(\pi \frac{t - \tau/2}{\tau/2} \right) \right), & \text{if } t \leq \tau, \\ 0, & \text{if } t > \tau. \end{cases} \quad (3)$$

2.2. Numerical method

The numerical simulations use the Chebyshev collocation method where the solution is approximated at gridpoints by a polynomial that is easy to differentiate. Unlike the finite difference methods, it is a global method where all the points contribute to the derivatives at each point. Its main advantage is lower computational cost due to the smaller number of points needed to describe the problem and simplicity of use in case of nonlinearities and a high order of spatial derivatives. For the Chebyshev collocation method the variables are stored at the Chebyshev extrema points, allowing the interpolation scheme to avoid the Runge's phenomenon (Fig. 5), which would arise in case of equidistant distribution of collocation points (Fig. 6) [15]. The spatial differentiation uses one Chebyshev differentiation matrix [16], and the integration in time is carried out using a `vode` solver [17] in the SciPy package [18].

The spatial differentiation and calculations are initially done on each layer separately. Thereafter the layers are interconnected by carrying over the stress and the particle velocity as shown in Fig. 7, allowing the energy to propagate both ways. The boundary conditions of stress $\sigma = 0$ or particle velocity $v = 0$ can be specified according to the problem.
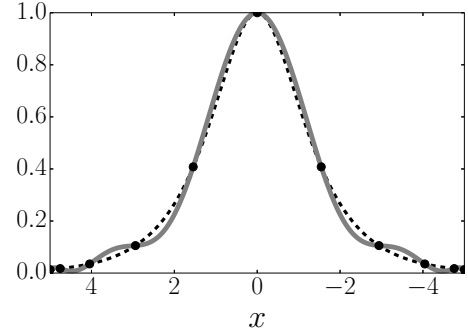

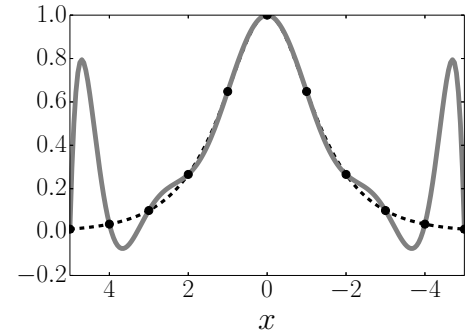**Fig. 5.** Interpolation in the case of Chebyshev points.

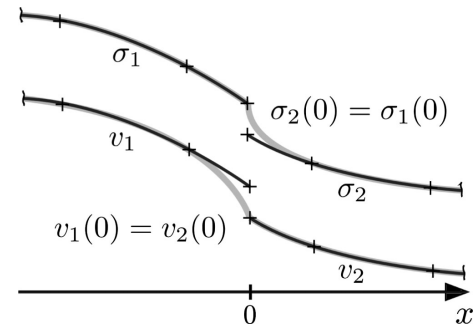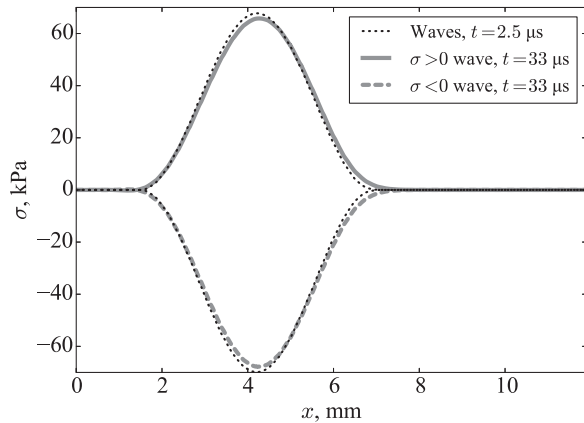**Fig. 6.** Interpolation in the case of equi-spaced points.

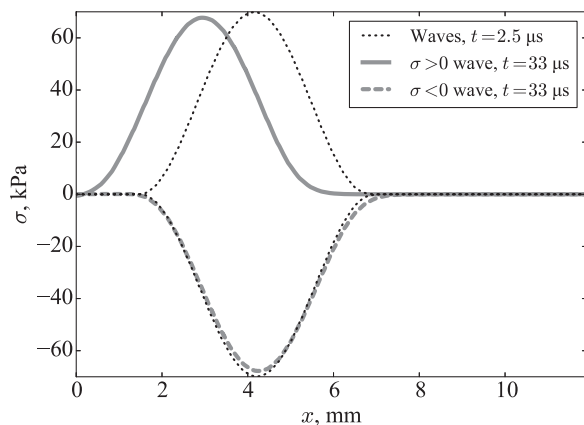**Fig. 7.** Matching between the layers during the timestep.

3. SIMULATIONS

Dynamic boundary conditions (Eq. (3)) were used to excite the wave in the medium. The simulation scheme was verified by doubling the number of spatial grid points, running the simulation again, and comparing the results. In case of little or no change in results, the scheme is suitable. For some material parameters, Eq. (1) with purely classical nonlinearity ($\alpha = 1$ in Eq. (2)) has been proven to sustain solitonic waves [19]. The simulations in this work are done to suggest the possibility of the existence of solitary waves in case of CFRP material parameters when introducing classical *and nonclassical* nonlinearities.

Firstly, a wide pulse half-cosine stress wave of Eq. (3), where $\tau = 2$ μs, was inserted into a 43-mm thick material. The pulse is allowed to reflect from the rear wall, return, and reflect from the front wall. The nearly initial pulse and twice-reflected pulse are compared. The reflections are, for the simplicity of analysis, from fixed ends. This means that the sign of the pulse is not changed by the reflections. The 'wavelength' corresponds to about 15 pairs of CFRP–epoxy layers. Figure 8 illustrates comparison between two cases: Fig. 8a where there is only classical nonlinearity $\beta = -15$ and $\alpha = 1$ for all ε; and Fig. 8b where there are both classical and nonclassical nonlinearities $\beta = -15$ and $\alpha = 97\%$ if $\varepsilon \geq 0$. The results do not exhibit an oscillatory tail behind the pulse (toward $x = 0$). In Fig. 8a the classical nonlinearity of $\beta = -15$ does not change

the propagation characteristics in any noticeable way. In Fig. 8b the addition of nonclassical nonlinearity $\alpha = 97\%$ decreases the velocity of the positive pulse.

Secondly, the initial pulse width was shortened to $\tau = 0.4$ μs, corresponding to about three pairs of epoxy–CFRP simulated laminate. The shorter wavelength 'feels' the microstructure and introduces oscillations due to the dispersion. The results are shown in Fig. 9. The case of of $\beta = -15$ and $\alpha = 1$ for all ε is shown in Fig. 9a, and for $\beta = -15$ and $\alpha = 97\%$ if $\varepsilon \geq 0$ in Fig. 9b. Obviously the velocity of the positive σ pulse is again lower than for the negative pulse. Additionally, the effect of the inhomogeneous medium is immediately recognizable by an oscillatory tail of the pulses. Furthermore, in case of nonclassical nonlinearity in Fig. 9b, both positive and negative pulses change shape in the propagation. For the positive pulse the oscillatory tail has increased slightly. For the negative pulse the oscillatory tail decreases and smoothens.

(a) Only classical nonlinearity: $\beta = -15$, $\alpha = 1$.

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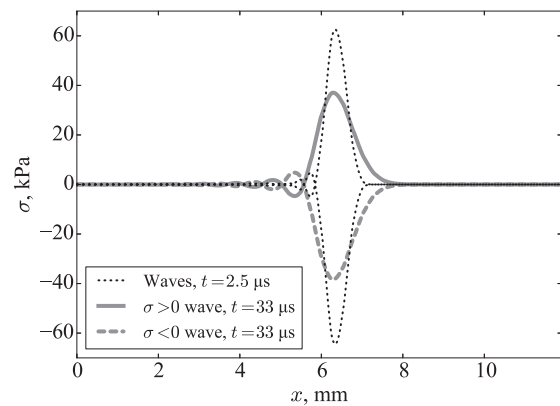

(b) Both nonlinearities: $\beta = -15$, $\alpha = 97\%$.
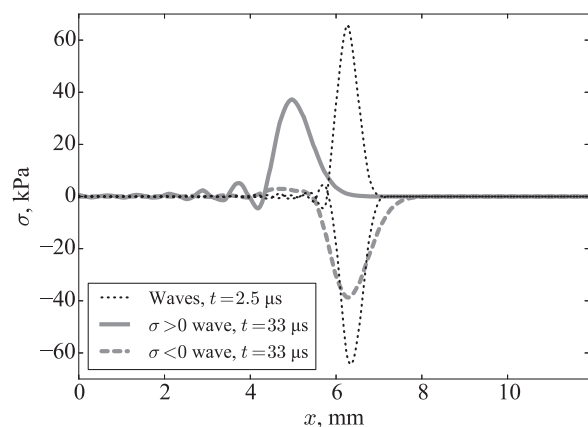

(b) Both nonlinearities: $\beta = -15$, $\alpha = 97\%$.

Fig. 8. Propagation of a half-cosine pulse with the width of $\tau = 2$ μs. The black dotted lines show the wave profile near the beginning of the propagation. Bold grey lines show wave profiles of positive and negative polarities after propagating and reflecting twice in the 43 mm wide medium. The spatial coordinate is denoted by x.

Fig. 9. Propagation of a half-cosine $\tau = 0.4$ μs pulse. The black dotted lines show the wave profile near the beginning of the propagation. Bold grey lines show wave profiles of positive and negative polarities after propagating and reflecting twice in the 43 mm wide medium. The spatial coordinate is denoted by x.

4. DISCUSSION

In the case of long-wavelength pulse $\tau = 2$ μs in Fig. 8, there is no noticeable wave steepening effect, which is normally found in nonlinear wave propagation. Since the pulses stay either purely positive or purely negative, the only nonlinearity affecting the *shapes* of the pulses is classical nonlinearity β, while the nonclassical nonlinearity α only affects the velocities of the pulses.

The situation changes when the pulse length is shortened to $\tau = 0.4$ μs, because it will become affected by the layered material, causing dispersion. The dispersion generates an oscillatory tail behind the main pulse for all results in Fig. 9. The purely classical nonlinearity with $\beta = -15$ (and $\alpha = 1$) is not strong enough to affect the wave propagation noticeably (Fig. 9a), only the dispersion decreases the amplitude of the main peak. However, the situation is different with small nonclassical nonlinearity of $\alpha = 97\%$ (Fig. 9b), as both positive and negative pulses change shape. The oscillatory tail decreases for the negative pulse and increases for the positive pulse, resembling the behaviour of a solitary wave. Figure 9b furthermore shows that at the beginning of the propagation the shape of the positive pulse is slightly more gradual than the shape of the negative pulse. It resembles the wave-steepening effect commonly seen in nonlinear wave propagation, suggesting that the negative pulse behaves in a solitary wave-like manner. Its speed of propagation is greater than that of the positive pulse and it is more stable thanks to the nonclassical nonlinearity counteracting the dispersion by affecting the positive parts of the oscillatory tail.

In these simulations the nonclassical nonlinearity $\alpha = 97\%$ affects the solution far more than the classical nonlinearity $\beta = -15$. The material parameters should be measured with nonlinear NDT techniques [20] in order to ascertain reasonable magnitudes for non-linearities. For measuring the nonclassical nonlinearity, a sinusoidal pulse could be propagated in this material. The pulse would have its negative part travelling faster than its positive. If the sinusoidal pulse was short (close to a single period), it would become compressed in propagation if the dynamic boundary condition was $\sigma(0,t) \sim +\sin$ and stretched if $\sigma(0,t) \sim -\sin$. The amount of distortion could indicate the magnitude of the nonclassical nonlinearity α in the constitutive Eq. (2).

5. CONCLUSIONS

It has been shown that in the case of large-wavelength pulses with the pulse width of 15 epoxy–CFRP pairs, the dispersion is not noticeably strong. Moreover, the classical nonlinearity of $\beta = -15$ with CFRP elasticity parameters is not strong enough to induce a noticeable change in wave shape. Introduction of nonclassical nonlinearity in addition to classical nonlinearity will bring about a speed difference between positive and negative pulses. The speed difference of positive and negative parts of a sinusoidal pulse here could indicate the magnitude of the nonlinearity.

However, a pulse with a length corresponding to 3–4 epoxy–CFRP layers will 'feel' the layered configuration of the material, so it will have an oscillatory tail due to the dispersion. Introducing a small *nonclassical* nonlinearity of the magnitude $\alpha = 97\%$ (in addition to classical nonlinearity $\beta = -15$) will change the shape of the pulse in different ways depending on the sign of the wave amplitude. Essentially, the oscillatory tail of the positive σ pulse will be increased and the tail of the negative σ pulse will be decreased. This resembles the propagation of a solitary wave by having (i) an effect resembling wave steepening, (ii) balancing between dispersion and nonlinearity, and (iii) a larger speed of a negative σ pulse compared to a positive pulse.

We found in this study that the nonclassical non-linearity would produce favourable effects for solitary wave propagation in the case of CFRP material, which could be used for the nonlinearity characterization and microdamage detection of the material. The nonclassical nonlinearity is zero-centred in this work and produces changes of velocity between the positive and negative parts of a wave even for small stress wave propagation. The material parameters, the type, and the magnitude of the nonlinearities need to be verified.

The Chebyshev collocation method was found suitable for 1D simulations of discontinuous media. The future work will include 2D simulations and analysis of wave propagation in complex nonlinear media. This should model potential materials better for solitary wave characterization and enable to take into account other complexities that surely affect the nonlinear acoustics of the material. As the results of this work show, it is necessary to consider additional sources of nonlinearity, other than the classical nonlinear parameter β, at different scales to see solitary wave-like evolution of waves.

Advances of imaging complex layered media by new signal processing schemes, involving solitonic coding, would improve the methods used today in medical imaging and NDT. Some biological complex layered media, such as the human skin, could benefit from such new coding schemes. Solitonic coding signal processing with using the orthogonality properties needed in classical nonlinear imaging potentially allows the use of elastic properties of soliton–soliton interactions in order to conduct fast nonlinear imaging.

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Üksiklainete leviku numbrilised simulatsioonid süsinikkiudkomposiitmaterjalis

Martin Lints, Andrus Salupere ja Serge Dos Santos

Üksiklaineteks nimetatakse dispersiivses ja mittelineaarses keskkonnas levivaid stabiilse kujuga lokaliseeritud laineid. Juhul kui üksiklained interakteeruvad omavahel elastselt, nimetatakse neid solitonideks. Üksiklained võivad mittepurustavas testimises osutuda kasulikeks tänu nende suurele stabiilsusele ja kuju sõltuvusele materjali omadustest. Nende rakendamine võib lubada keerulise geomeetria või keerulise sisestruktuuriga materjalide (mikrostruktuur- või granulaarsed materjalid) detailsemat või kiiremat uurimist.

Antud töös on uuritud süsinikkiudkomposiiti kui materjali, millel võivad olla nii klassikalised kui ka (mikro-kahjustustest tulenevalt) mitteklassikalised mittelineaarsused. Dispersiooni tekitab keskkonna perioodiline kihilisus. Modelleeritav materjal on 144-kihiline süsinikkiudkomposiit. Antud töös oli kasutusel lihtne laminaatmudel tükati pidevatest keskkondadest. Matemaatiline mudel baseerub Cauchy liikumisvõranditel ja mittelineaarsel olekuvõrrandil, kus materjali jäikus võib sõltuda nii deformatsiooni suurusest kui ka selle märgist. Numbrilised eksperimendid on tehtud Chebyshevi pseudospektraalmeetodiga.

Simulatsioonide tulemused näitavad, et kuigi klassikaline mittelineaarsus võib olla liiga väike, tasakaalustamaks dispersiooni simuleeritud komposiidis, siis seevastu üsna väike mitteklassikaline mittelineaarsus muudab lainelevikut olulisel määral. Sealjuures on sellisel mittelineaarsusel suuremate lainepikkuste korral ilmne efekt positiivse ja negatiivse pingelaine leviku kiiruses. Väiksemate lainepikkuste juures, kus dispersioon avaldub tugevamalt, on positiivse ja negatiivse amplituudiga lainetel lisaks liikumiskiiruse erinevusele ka oluline erinevus laine kujus ning selle "sabas" olevate ostsillatsioonide suuruses. Selline negatiivne pingelaine on suhteliselt stabiilne ja selle omadused sarnanevad üksiklaine omadustele, mis viitab mitteklassikalise mittelineaarsuse soodsale mõjule üksiklainete tekitamisel kihilistes materjalides.