Proceedings of the Estonian Academy of Sciences,
2015, **64**, 2, 127–138
doi: 10.3176/proc.2015.2.01
Available online at www.eap.ee/proceedings

MATHEMATICS

Effect of structural parameters on the vibrational response of a visco-elastic rectangular plate with clamped ends

Anupam Khanna[a*] and Narinder Kaur[b]

[a] Department of Mathematics, DAV College, Sadhaura, Yamunanagar, Haryana, India
[b] Department of Mathematics, Maharishi Markandeshwar University-Mullana, Ambala, Haryana, India

Received 14 July 2014, revised 26 November 2014, accepted 8 December 2014, available online 22 May 2015

Abstract. A mathematical model for predicting the vibrational response of a non-homogeneous visco-elastic rectangular plate was developed to assist design engineers and researchers. In the presented model, thermally induced vibrations of a four-sided clamped rectangular plate of non-uniform thickness is discussed. Non-homogeneity in the material is characterized exponentially in Poisson's ratio while temperature variation is considered bi-parabolic. A boundary value fourth order partial differential equation of motion is formulated for the parabolic tapered rectangular plate. Visco-elastic properties of the material are of Kelvin type, and deflection is considered small and linear. This paper focuses on the effects of structural parameters, i.e. thermal gradient, taper constant, aspect ratio, and non-homogeneity constant on the vibrational behaviour of rectangular plates. The Rayleigh–Ritz method is used to obtain results for the time period and deflection for the first two modes of vibration. Comparison of the results of the present paper with others available in the literature is visualized with the help of graphs.

Key words: visco-elastic, non-homogeneous, structural parameters, deflection, thermal gradient.

Nomenclature

a	– length of the rectangular plate, m	$\tilde{D}$	– visco-elastic operator
b	– breadth of the rectangular plate, m	ρ	– density of the plate material, kg/m^3
x, y	– co-ordinates in the plane of the plate	t	– time, s
h	– thickness of the plate, m	η	– visco-elastic constant, N·s/m^2
M_x, M_y	– bending moments, N·m	$w(x, y, t)$	– deflection, m
M_{xy}	– twisting moment, N·m	$\phi(x, y)$	– deflection function, m
E	– Young's modulus, N/m^2	$\psi(t)$	– time function, s
G	– shear modulus, N/m^2	β	– taper constant
ν	– Poisson's ratio	α	– thermal gradient
D_1	– flexural rigidity, N·m	α_1	– non-homogeneity constant

1. INTRODUCTION

The designs of complex structures, i.e. jet engines, helicopter yokes, submarines, etc., are based on the vibrational analysis of the structural system. Due to the variability in the mechanical prospective of the structure, the vibrational behaviour of the structure is affected. Therefore, it becomes necessary to analyse the behaviour of structural systems for the preliminary phase of designing structures so that their hydrodynamic performance and stability during navigation and operations can be significantly improved.

[*] Corresponding author, rajieanupam@gmail.com

Tapered visco-elastic plates are commonly used in various engineering applications and structures, e.g. the aerospace industry, missiles, etc., under the influence of elevated temperature. Since elevated temperature directly affects the mechanical properties of the material of plates, the effect of temperature variations can not be neglected. Hence, active vibration control in engineering structures and systems has attracted the attention of mathematical physicists and design engineers for many years.

Abu et al. [1] discussed two dimensional transient wave propagation in visco-elastic layered media. Avalos and Laura [2] treated transverse vibrations of a simply supported plate of generalized anisotropy with oblique cut-outs. Hasheminejad et al. [4] discussed the exact solution for free vibrations of an eccentric elliptical plate. Hosseini-Hashemi et al. [5] made a mathematical study on the free vibration of stepped thickness circular/annular Mindlin functionally graded plates. Khanna and Arora [7] discussed thermally induced vibration of a tapered parallelogram plate with mixed boundary conditions. Khanna and Sharma [10] obtained natural vibration of a visco-elastic plate of varying thickness with the thermal effect. Lal and Kumar [11] evaluated transverse vibrations of non-homogeneous rectangular plates with bi-linear thickness variation. Lal et al. [12] discussed transverse vibrations of non-homogeneous rectangular plates of uniform thickness using boundary characteristic orthogonal polynomials. Leissa [13] provided a collection of research papers on the vibration of plates of various shapes and size in his monograph. Tariverdilo et al. [16] analysed asymmetric free vibration of a coupled system including a clamped circular plate in contact with an incompressible bounded fluid. Sharma et al. [15] worked on the effect of pasternak foundation on the axisymmetric vibration of polar orthotropic annular plates of varying thickness. Wang and Chen [17] discussed the axisymmetric vibration and damping analysis of rotating annular plates with constrained damping layer treatments.

In the present model, the authors analysed the vibrational behaviour of a non-homogeneous rectangular plate. It is assumed that the thickness of the rectangular plate varies parabolically in x-direction. The Rayleigh–Ritz technique is used to determine the frequency equation of the plate. The time period and deflection at different points for the first two modes of vibration are obtained for various values of structural parameters.

2. GEOMETRY OF THE PLATE

In the present study, a rectangular plate of varying thickness in one direction $h(x)$, exponential varying Poisson's ratio $\nu(x)$, and constant density ρ is investigated as shown in Fig. 1. Plate OABC is assumed to be placed in such a way that point O is the origin of the xy-plane and sides OA and OC overlap on x- and y-axis, respectively. The domain of the plate in the xy-plane is $0 \le x \le a$ and $0 \le y \le b$, where a and b are the length and breadth of the plate, respectively.

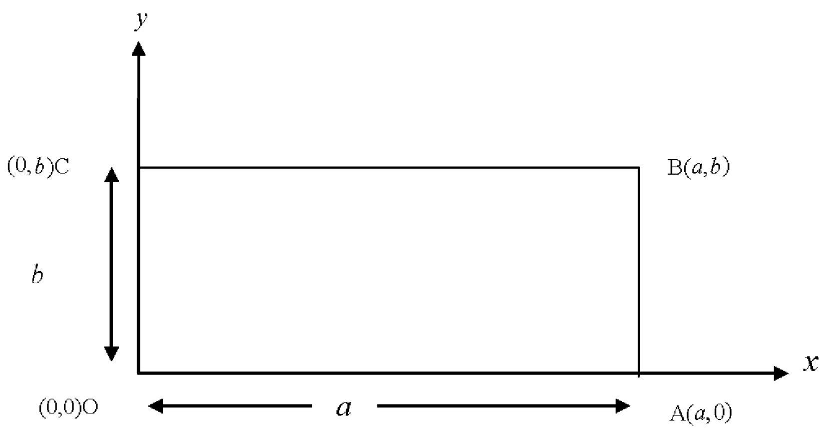


Fig. 1. Rectangular plate in Cartesian coordinate (xy-plane).

3. DIFFERENTIAL EQUATION OF MOTION

The equation of motion of a visco-elastic plate of variable thickness is [6]

$$M_{x,xx} + 2M_{xy,xy} + M_{y,yy} = \rho h w_{,tt}, \tag{1}$$

where

$$M_x = -\tilde{D}D_1(w_{,xx} + \nu w_{,yy}),\quad M_y = -\tilde{D}D_1(w_{,yy} + \nu w_{,xx}),\quad M_{xy} = -\tilde{D}D_1(1-\nu)w_{,xy}. \tag{2}$$

Here, ‘,’ denotes partial differentiation with respect to the following suffix.

Using Eq. (2) in Eq. (1), one gets [3]

$$\tilde{D}[\nabla^2(D_1\nabla^2 w) - (1-\nu)\Diamond^4(D_1, w)] + \rho h\frac{\partial^2 w}{\partial t^2} = 0, \tag{3}$$

where $\Diamond^4(D_1, w) = \frac{\partial^2 D_1}{\partial y^2}\frac{\partial^2 w}{\partial x^2} - 2\frac{\partial^2 D_1}{\partial x\partial y}\frac{\partial^2 w}{\partial x\partial y} + \frac{\partial^2 D_1}{\partial x^2}\frac{\partial^2 w}{\partial y^2}$ is the die operator and $\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is the Laplacian operator.

The solution of Eq. (3) can be assumed as

$$w(x, y, t) = \phi(x, y)\psi(t). \tag{4}$$

Substituting Eq. (4) into Eq. (3), one obtains

$$\left[D_1\left(\frac{\partial^4\phi}{\partial x^4} + 2\frac{\partial^4\phi}{\partial x^2\partial y^2} + \frac{\partial^4\phi}{\partial y^4}\right) + 2\frac{\partial D_1}{\partial x}\left(\frac{\partial^3\phi}{\partial x^3} + \frac{\partial^3\phi}{\partial x\partial y^2}\right) + 2\frac{\partial D_1}{\partial y}\left(\frac{\partial^3\phi}{\partial y^3} + \frac{\partial^3\phi}{\partial y\partial x^2}\right)\right.$$
$$\left. + 2\frac{\partial^2 D_1}{\partial x^2}\left(\frac{\partial^2\phi}{\partial x^2} + \nu\frac{\partial^2\phi}{\partial y^2}\right) + 2\frac{\partial^2 D_1}{\partial y^2}\left(\frac{\partial^2\phi}{\partial y^2} + \nu\frac{\partial^2\phi}{\partial x^2}\right) + 2(1-\nu)\frac{\partial^2 D_1}{\partial x\partial y}\frac{\partial^2\phi}{\partial x\partial y}\right] \Big/ \rho h\phi = -\ddot{\psi} / \tilde{D}\psi, \tag{5}$$

where the dots denote differentiation with respect to t.

Both sides of the preceding equation are independent. Therefore, it is satisfied if both sides are equal to a positive constant, i.e. ξ^2. In this case, one gets

$$\left[D_1\left(\frac{\partial^4\phi}{\partial x^4} + 2\frac{\partial^4\phi}{\partial x^2\partial y^2} + \frac{\partial^4\phi}{\partial y^4}\right) + 2\frac{\partial D_1}{\partial x}\left(\frac{\partial^3\phi}{\partial x^3} + \frac{\partial^3\phi}{\partial x\partial y^2}\right) + 2\frac{\partial D_1}{\partial y}\left(\frac{\partial^3\phi}{\partial y^3} + \frac{\partial^3\phi}{\partial y\partial x^2}\right)\right.$$
$$\left. + 2\frac{\partial^2 D_1}{\partial x^2}\left(\frac{\partial^2\phi}{\partial x^2} + \nu\frac{\partial^2\phi}{\partial y^2}\right) + 2\frac{\partial^2 D_1}{\partial y^2}\left(\frac{\partial^2\phi}{\partial y^2} + \nu\frac{\partial^2\phi}{\partial x^2}\right) + 2(1-\nu)\frac{\partial^2 D_1}{\partial x\partial y}\frac{\partial^2\phi}{\partial x\partial y}\right] - \rho\xi^2 h\phi = 0 \tag{6}$$

and

$$\ddot{\psi} + \xi^2\tilde{D}\psi = 0. \tag{7}$$

Eq. (6) and Eq. (7) are differential equations of motion and time function for a visco-elastic rectangular plate, respectively.

Here, D_1 is flexural rigidity of a rectangular plate [14], i.e.

$$D_1 = \frac{Eh^3}{12(1-\nu^2)}. \tag{8}$$

4. LIMITATIONS

Assessment of vibrations of structures is very much complicated due to the variability in structural parameters. Therefore, some restrictions are required for predicting the vibration response of structures. The authors assumed the following limitations in the present model for further investigation:

(i) Most structures are usually worked in the presence of high temperature. Temperature distributions in any structural system vary from point to point. Due to variations in the temperature field, vibrational properties of the structure vary significantly. So, it becomes necessary to analyse the temperature effect on the vibrations of structures or systems.

In the present model, the authors assumed bi-parabolic variation in the temperature field as follows:

$$\tau = \tau_0 \left(1 - \frac{x^2}{a^2}\right)\left(1 - \frac{y^2}{b^2}\right). \tag{9}$$

The temperature dependence of the modulus of elasticity for most engineering materials can be expressed as:

$$E = E_0 (1 - \tau\gamma), \tag{10}$$

where E_0 is the value of Young's modulus at reference temperature, i.e. $\tau = 0,$ and γ is the slope of the variation of E with $\tau.$ After substituting τ form Eq. (9), Eq. (10) becomes

$$E = E_0 \left[1 - \alpha \left(1 - \frac{x^2}{a^2}\right)\left(1 - \frac{y^2}{b^2}\right)\right], \tag{11}$$

where $\alpha = \gamma\tau_0.$

(ii) To fulfil the twofold requirement of safety and economy, plates of variable thickness are commonly used in engineering applications, e.g. to make blades of turbines, bridge plates, fins of plane, etc. In order to investigate the effect of tapering on the vibrational behaviour of plates, the authors assumed parabolic tapering in this model:

$$h = h_0 \left(1 + \beta \frac{x^2}{a^2}\right), \tag{12}$$

where h_0 is the thickness of the plate at $x = y = 0.$

(iii) Physical properties of materials, i.e. their density, Poisson's ratio, etc., vary point to point in case of non-homogeneity is present in the material. In the present model, the authors assumed exponential variation in Poisson's ratio as

$$\nu = \nu_0 e^{\alpha_1 \frac{x}{a}}, \tag{13}$$

where ν_0 denotes Poisson's ratio at reference temperature and α_1 is non-homogeneity constant. Since the maximum value of Poisson's ratio is less than 0.5, the numerical value of α_1 (as it varies exponentially in this paper) can not be greater than 0.16 (approximately). Hence, the variation in Poisson's ratio is taken from 0.0 to 0.15 (at most) in the calculation.

On using Eqs (11), (12), and (13) in Eq. (8), one obtains

$$D_1 = \frac{\left\{E_0\left[1-\alpha\left(1-\frac{x^2}{a^2}\right)\left(1-\frac{y^2}{b^2}\right)\right]\right\}\left[h_0\left(1+\beta\frac{x^2}{a^2}\right)\right]^3}{12\left(1-\nu_0^2 e^{2\alpha_1\frac{x}{a}}\right)}. \tag{14}$$

(iv) The ends of the visco-elastic rectangular plate are supposed to be clamped. Due to the clamped boundary, the following conditions must be satisfied by deflection function $\phi(x, y)$:

$$\left.\begin{array}{l}\phi = \phi_{,x} = 0,\ x = 0, a\\ \phi = \phi_{,y} = 0,\ y = 0, b\end{array}\right\}. \tag{15}$$

Also, the corresponding two-term deflection function $\phi(x, y)$ is taken as [8]

$$\phi(x, y) = \phi_1(x, y)\phi_2(x, y), \tag{16}$$

where $\phi_1(x,y) = \left[\frac{x}{a}\frac{y}{b}\left(1-\frac{x}{a}\right)\left(1-\frac{y}{b}\right)\right]^2$, and $\phi_2(x,y) = \zeta_1 + \zeta_2\frac{x}{a}\frac{y}{b}\left(1-\frac{x}{a}\right)\left(1-\frac{y}{b}\right)$,

where ζ_1 and ζ_2 are arbitrary constants.

Here $\phi_1(x, y)$ satisfies all boundary conditions and $\phi_2(x, y)$ provides two modes of vibration.

5. SOLUTION OF THE DIFFERENTIAL EQUATION OF MOTION

To obtain a frequency equation for the vibration of the rectangular plate, the authors used the Rayleigh–Ritz method. This method is based on the principle of conservation of energy, i.e. the maximum strain energy P must be equal to the maximum kinetic energy K. So, it is necessary for the problem under consideration that [8]

$$\delta(P-K) = 0, \tag{17}$$

where

$$K = (0.5)\rho\xi^2\int_0^a\int_0^b h\phi^2\,dydx \tag{18}$$

and

$$P = (0.5)\int_0^a\int_0^b D_1\left\{\left(\frac{\partial^2\phi}{\partial x^2}\right)^2 + \left(\frac{\partial^2\phi}{\partial y^2}\right)^2 + 2\nu\frac{\partial^2\phi}{\partial x^2}\frac{\partial^2\phi}{\partial y^2} + 2(1-\nu)\left(\frac{\partial^2\phi}{\partial x\partial y}\right)^2\right\}dydx. \tag{19}$$

To simplify and parameterize the present problem, non-dimensionalization is introduced as

$$X = \frac{x}{a},\quad Y = \frac{y}{a}. \tag{20}$$

After using Eq. (20) in Eq. (18) and Eq. (19), one gets

$$K^* = (0.5)\rho\xi^2 a^2 h_0\int_0^1\int_0^{b/a}(1+\beta X^2)\phi^2\,dYdX \tag{21}$$

and

$$P^* = Q\int_0^1\int_0^{b/a}\left\{\begin{aligned}&\left\{\frac{\left[1-\alpha(1-X^2)\left(1-\frac{a^2}{b^2}Y^2\right)\right](1+\beta X^2)^3}{\left(1-\nu_0^2 e^{2\alpha_1 X}\right)}\right\}\\&\times\left\{\left(\frac{\partial^2\phi}{\partial X^2}\right)^2+\left(\frac{\partial^2\phi}{\partial Y^2}\right)^2+2\nu_0 e^{\alpha_1 X}\frac{\partial^2\phi}{\partial X^2}\frac{\partial^2\phi}{\partial Y^2}+2\left(1-\nu_0 e^{\alpha_1 X}\right)\left(\frac{\partial^2\phi}{\partial X\partial Y}\right)^2\right\}\end{aligned}\right\}dYdX, \tag{22}$$

where $Q = \frac{E_0 h_0^3}{24a^2}$.

Using Eq. (21) and Eq. (22) in Eq. (17), one obtains

$$(P_E^* - \lambda^2 K_E^*) = 0. \tag{23}$$

Here, $\lambda^2 = \frac{12\rho\xi^2 a^4}{E_0 h_0^2}$ is the frequency parameter. Equation (23) contains two unknown constants, i.e. ζ_1 and ζ_2, arising due to the substitution of $\phi(x, y)$. These two constants are to be determined as follows:

$$\frac{\delta(P_E^* - \lambda^2 K_E^*)}{\delta\zeta_n} = 0, \qquad n = 1, 2. \tag{24}$$

On simplifying Eq. (24), one gets

$$C_{n1}\zeta_1 + C_{n2}\zeta_2 = 0, \qquad n = 1,2, \tag{25}$$

where C_{n1}, C_{n2} for $n = 1, 2$ involve structural parameters and the frequency parameter λ^2.

Equation (25) is a set of two simultaneous homogeneous equations with variables ζ_1 and ζ_2 having an infinite number of solutions. If one choose $\zeta_1 = 1$, one can easily evaluate ζ_2, i.e.

$$\zeta_2 = \frac{-C_{n1}}{C_{n2}}.$$

For a non-trivial solution, the determinant of the coefficients of Eq. (25) must be zero, i.e.

$$\begin{vmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{vmatrix} = 0. \tag{26}$$

Equation (26) is a quadratic equation in λ^2 from which two values of λ^2 can be extracted.

The time period of the vibration of the visco-elastic plate is given by

$$K = \frac{2\pi}{\lambda}. \tag{27}$$

6. SOLUTION FOR THE TIME FUNCTION $\psi(t)$

Time functions of free vibration of visco-elastic plates are defined by the general ordinary differential Eq. (7). Its form depends on the visco-elastic operator $\tilde{D}$.

For Kelvin's model, one gets [6]

$$\tilde{D} \equiv 1 + \frac{\eta}{G}\frac{d}{dt}. \tag{28}$$

After using Eq. (28) in Eq. (7), Eq. (7) is modified as follows:

$$\ddot{\psi} + \xi^2\left(1 + \frac{\eta}{G}\frac{d}{dt}\right)\psi = 0,$$

$$\ddot{\psi} + \xi^2\frac{\eta}{G}\dot{\psi} + \xi^2\psi = 0. \tag{29}$$

Now, Eq. (29) is a differential equation of order two with respect to t for the time function ψ. The solution of Eq. (29) can be obtained as [6]

$$\psi(t) = e^{a_1 t}(C_1 \cos b_1 t + C_2 \sin b_1 t), \tag{30}$$

where

$$a_1 = \frac{-\xi^2\eta}{2G}, \qquad b_1 = \xi\sqrt{1 - \left(\frac{\xi\eta}{2G}\right)^2},$$

and C_1 and C_2 are constants which can be determined easily from the initial conditions of the plate.

Let us take the initial conditions as

$$\psi = 1 \text{ at } t = 0 \tag{31}$$

and

$$\dot{\psi} = 0 \text{ at } t = 0. \tag{32}$$

Substituting Eq. (31) in Eq. (30), one gets

$$C_1 = 1. \tag{33}$$

After using Eq. (32) in Eq. (30), one obtains

$$C_2 = \frac{-a_1}{b_1}. \tag{34}$$

Substituting C_1 and C_2 from Eq. (33) and Eq. (34) in Eq. (30), Eq. (30) becomes

$$\psi(t) = e^{a_1 t}\left[\cos b_1 t + \left(\frac{-a_1}{b_1}\right)\sin b_1 t\right]. \tag{35}$$

7. FORMULATION OF DEFLECTION

With the help of the values of ζ_1 and ζ_2, one can obtain deflection function ϕ as

$$\phi = \left[XY\frac{a}{b}(1-X)\left(1-\frac{a}{b}Y\right)\right]^2 \times \left[1 + \left(\frac{-C_{11}}{C_{12}}\right)XY\frac{a}{b}(1-X)\left(1-\frac{a}{b}Y\right)\right]. \tag{36}$$

On using Eq. (35) and Eq. (36) in Eq. (4), one gets

$$w=\left\{\left[XY\frac{a}{b}(1-X)\left(1-\frac{a}{b}Y\right)\right]^2\times\left[1+\left(\frac{-C_{11}}{C_{12}}\right)XY\frac{a}{b}(1-X)\left(1-\frac{a}{b}Y\right)\right]\right\}\times\left\{e^{a_1t}\left[\cos b_1t+\left(-\frac{a_1}{b_1}\right)\sin b_1t\right]\right\}. \tag{37}$$

8. RESULTS AND DISCUSSION

In calculations, the following parameters are used for duralumin (an alloy of aluminium) [9]:

$$E_0=7.08\times10^{10}\,\frac{N}{\text{m}^2},\quad G=2.632\times10^{10}\,\frac{N}{\text{m}^2},\quad \eta=14.612\times10^{5}\,\frac{Ns}{\text{m}^2},\quad \rho=2.80\times10^{3}\,\frac{\text{kg}}{\text{m}^3},$$
$$\upsilon_0=0.345,\quad h_0=0.01\,\text{m}.$$

Table 1 presents the time period for the first two modes of vibration with a fixed aspect ratio $(a/b=1.5)$ for different values of the non-homogeneity constant α_1 (i.e. $\alpha_1=0.0, 0.05, 0.10, 0.15$) at the following combinations of the thermal gradient α and the taper constant β:

$$\alpha=\beta=0.0;\quad \alpha=\beta=0.2;\quad \alpha=\beta=0.6;\quad \alpha=\beta=0.8.$$

It is interesting to note that the time period for the first two modes of vibration decreases with increasing α_1 for all combinations of α and β. As the combined value of α and β increases from 0.0 to 0.8, the time period also decreases for both modes of vibration.

Table 2 tabulates the time period for the first two modes of vibration for fixed values of the thermal gradient and the taper constant, i.e. $\alpha=\beta=0.2$, for different values of the non-homogeneity constant α_1 and the aspect ratio a/b. Here, the authors found a continuous decrement in the time period (for both modes) as α_1 increases from 0.0 to 0.15 and a/b increases simultaneously from 0.25 to 1.5.

Table 1. Time period vs non-homogeneity constant at a/b = 1.5

α_1	$\alpha=\beta=0.0$		$\alpha=\beta=0.2$		$\alpha=\beta=0.6$		$\alpha=\beta=0.8$	
	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2
0.0	668.23	169.09	658.30	167.11	641.08	163.35	635.91	161.86
0.05	665.99	168.52	656.07	166.52	638.91	162.73	633.80	161.22
0.10	663.62	167.92	653.70	165.90	636.59	162.06	631.52	160.53
0.15	661.11	167.28	651.17	165.24	634.09	161.35	629.06	159.79

Table 2. Time period vs aspect ratio at $\alpha=\beta=0.2$

a/b	$\alpha_1=0.0$		$\alpha_1=0.05$		$\alpha_1=0.10$		$\alpha_1=0.15$	
	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2
0.25	1705.27	416.69	1698.59	415.00	1691.35	413.17	1683.48	411.16
0.5	1585.19	395.12	1579.16	393.56	1572.64	391.85	1565.57	390.00
0.75	1367.51	347.39	1362.52	346.06	1357.15	344.62	1351.35	343.06
1.0	1100.30	281.61	1096.44	280.58	1092.30	279.47	1087.86	278.27
1.25	853.26	217.90	850.33	217.13	847.22	216.30	843.89	215.41
1.5	658.30	167.11	656.07	166.52	653.70	165.90	651.17	165.24

For $0 \le X \le 1, 0 \le Y \le 1$ the deflection function $\phi(x, y)$ assumed in Eq. (16) shows the following properties:

$$\phi\left(X, \frac{a}{b}Y\right) = \phi\left(1 - X, \frac{a}{b}Y\right),$$

$$\phi\left(X, \frac{a}{b}Y\right) = \phi\left(X, 1 - \frac{a}{b}Y\right),$$

$$\phi\left(X, \frac{a}{b}Y\right) = \phi\left(1 - X, 1 - \frac{a}{b}Y\right).$$

Therefore, it has the same values for $X = 0.2$ and $X = 0.8$ as well as $X = 0.4$ and $X = 0.6$. The same is valid for $(a/b)Y$. Also for $X = 0.0$ and $X = 1.0$ or $Y = 0.0$ and $Y = 1.0$, deflection becomes zero. The deflection for both modes of vibration is reported at different values of X and Y for various values of plate parameters in Tables 3–5 as follows:

Table 3: $\alpha = \beta = 0.0$; $a/b = 1.5$; $\alpha_1 = 0.0, 0.05, 0.10, 0.15$; $\psi = 0K, 5K$.
Table 4: $\alpha = \beta = 0.6$; $a/b = 1.5$; $\alpha_1 = 0.0, 0.05, 0.10, 0.15$; $\psi = 0K, 5K$.
Table 5: $\alpha = \beta = 0.2$; $\alpha_1 = 0.1$; $a/b = 0.25, 0.50, 0.75, 1.0, 1.25, 1.5$; $\psi = 0K, 5K$.

In Table 3, both modes of deflection (at $\psi = 0K$ and $\psi = 5K$) continuously decrease as the non-homogeneity constant α_1 increases for each paired value of X and Y. Also, it is interesting to note that the values of deflection for both modes of vibration are greater at $\psi = 0K$ as compared to $\psi = 5K$.

In Table 4, deflection shows different variations for $\psi = 0K$ and $\psi = 5K$. At $\psi = 0K$, deflection for both modes of vibration continuously increases as the non-homogeneity constant α_1 increases for each paired value of X and Y. At $\psi = 5K$, the first mode of deflection continuously decreases but the second mode of deflection continuously increases as the non-homogeneity constant α_1 increases for each paired value of X and Y.

Variation in deflection for both modes of vibration for various values of the aspect ratio can be explained with the help of Table 5. A rapid increment is found in deflection for both modes of vibration as the aspect ratio a/b increases for each paired value of X and Y at $\psi = 0K$ and $\psi = 5K$.

Table 3. Deflection ($\times 10^{-5}$) vs the non-homogeneity constant for $\alpha = \beta = 0.0$; $a/b = 1.5$ at $\psi = 0K$ and $\psi = 5K$ (in parentheses)

α_1	X / Y	0.2		0.4	
		Mode 1	Mode 2	Mode 1	Mode 2
0.0	0.2	114.6210 (50.4673)	39.5098 (1.8094)	259.8370 (114.3570)	6.3376 (0.2902)
0.05		114.6330 (50.3336)	39.5099 (1.8036)	259.8770 (114.1080)	6.3378 (0.2893)
0.10		114.6360 (50.1875)	39.5100 (1.7987)	259.8900 (113.7790)	6.3383 (2.8856)
0.15		114.6450 (50.0341)	39.5103 (1.7939)	259.9190 (113.4350)	6.3393 (2.8784)
0.0	0.6	20.8718 (9.1859)	14.9592 (0.6847)	47.1142 (20.7443)	27.1595 (1.2435)
0.05		20.8727 (9.1649)	14.9593 (0.6830)	47.1174 (20.6886)	27.1595 (1.2400)
0.10		20.8730 (9.1381)	14.9593 (0.6810)	47.1184 (20.6283)	27.1595 (1.2364)
0.15		20.8737 (9.1098)	14.9593 (0.6792)	47.1207 (20.5647)	27.1596 (1.2331)

Table 4. Deflection ($\times 10^{-5}$) vs the non-homogeneity constant for $\alpha = \beta = 0.6$; $a/b = 1.5$ at $\psi = 0K$ and $\psi = 5K$ (in parentheses)

α_1	X \ Y	0.2		0.4	
		Mode 1	Mode 2	Mode 1	Mode 2
0.0	0.2	126.2400 (53.6841)	39.5098 (1.7888)	299.0520 (127.1730)	6.3376 (0.3114)
0.05		126.3370 (53.5692)	39.6812 (1.7898)	299.3780 (126.9420)	6.9159 (0.3119)
0.10		126.4450 (53.4472)	39.6836 (1.7921)	299.7450 (126.6990)	6.9243 (0.3127)
0.15		126.5740 (53.3206)	39.6865 (1.7958)	300.1800 (126.4540)	6.9339 (0.3137)
0.0	0.6	21.7864 (9.2647)	14.9592 (0.6750)	50.2011 (21.3482)	27.1595 (1.2264)
0.05		21.7940 (9.2410)	14.9729 (0.6753)	50.2268 (21.2971)	27.2050 (1.2271)
0.10		21.8026 (9.2157)	14.9729 (0.6761)	50.2556 (20.2426)	27.2056 (1.2286)
0.15		20.8127 (9.1888)	14.9732 (0.6775)	50.2899 (21.1851)	27.2064 (1.2311)

Table 5. Deflection ($\times 10^{-5}$) vs the aspect ratio for $\alpha = \beta = 0.2$; $\alpha_1 = 0.1$ at $\psi = 0K$ and $\psi = 5K$ (in parentheses)

a/b	X \ Y	0.2		0.4	
		Mode 1	Mode 2	Mode 1	Mode 2
0.25	0.2	5.9672 (4.3158)	4.9304 (1.3069)	13.6413 (9.8662)	10.1422 (2.6884)
0.5		21.6542 (15.2833)	14.9759 (3.6924)	49.7548 (35.1165)	27.2157 (6.7102)
0.75		43.0981 (28.7808)	25.2114 (5.1270)	98.6381 (65.8702)	38.2705 (7.7828)
1.0		67.2904 (40.7444)	33.0856 (4.6322)	153.3770 (92.8699)	37.9359 (5.3113)
1.25		92.5818 (48.4822)	37.7701 (2.9799)	211.2140 (110.6060)	26.2240 (2.0690)
1.5		117.2980 (50.7121)	39.5381 (1.7872)	268.8740 (116.2440)	6.4331 (0.2907)

9. COMPARISON AND CONCLUSIONS

The frequency for both modes of vibration in the present paper is compared with the frequency in [8] at the corresponding values of structural parameters for two cases:

Case (i) $\beta = 0.0; \alpha_1 = 0.1; \alpha = 0.0, 0.2, 0.4, 0.6, 0.8.$

Case (ii) $\alpha = 0.0; \alpha_1 = 0.1; \beta = 0.0, 0.2, 0.4, 0.6, 0.8.$

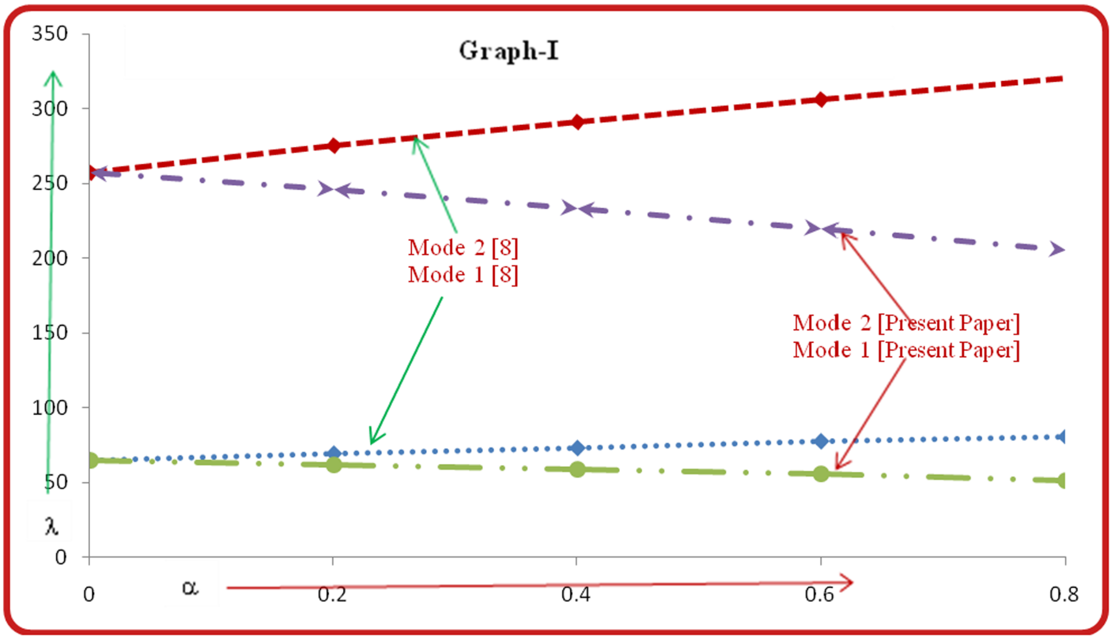
A graphical representation is provided for both the above cases (Fig. 2, Graph I for case (i) and Graph II for case (ii)). Graph I clearly shows that the frequency in the present paper is less than the frequency in [8] for both modes of vibration. But in Graph II, both modes of frequency in the present paper are greater than in [8].

The frequencies in the present paper and in [8] coincide only at $\alpha = \alpha_1 = \beta = 0.0$.

Based on the above graphical comparison, the authors conclude the following:

(i) In case of bi-parabolic variation in temperature, low frequency vibrations comparable to [8] can be obtained.

(ii) Tapering directly affects the vibrational behaviour of structures or plates. Frequency is higher in case of parabolic tapering (present paper) as compared to exponential tapering [8].



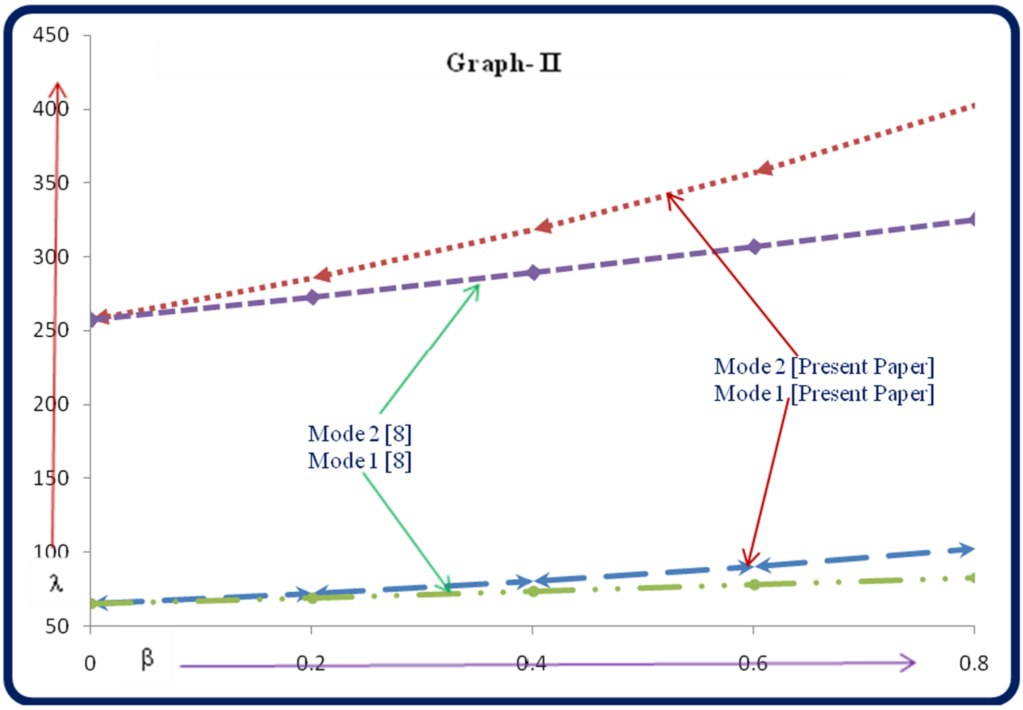


Fig. 2. Comparison of the frequencies of the two modes in the present study and in [8]. Graph I: $\beta = 0.0$; Graph II: $\alpha = 0.0$.

(iii) Frequency can be controlled actively by using appropriate values of structural parameters.

(iv) The present study confirms previous findings and also provides futuristic numerical data for researchers and design engineers to enhance the efficiency and reliability of machines and mechanical structures.

REFERENCES

1. Abu, A. I., Turhan, D., and Mengi, D. Two dimensional transient wave propagation in visco-elastic layered media. *J. Sound Vib.*, 2001, **244**(5), 837–858.
2. Avalos, D. R. and Laura, P. A. Transverse vibrations of a simply supported plate of generalized anisotropy with an oblique cut-out. *J. Sound Vib.*, 2002, **258**(2), 773–776.
3. Chakraverty, S. *Vibrations of Plate*. Vol. 10. Taylor and Francis, 2009.
4. Hasheminejad, S. M., Ghaheri, A., and Vaezian, S. Exact solution for free in-plane vibration analysis of an eccentric elliptical plate. *Acta Mech.*, 2013, **224**(8), 1609–1624.
5. Hosseini-Hashemi, S., Derakhshani, M., and Fadaee, M. An accurate mathematical study on the free vibration of stepped thickness circular/annular Mindlin functionally graded plates. *Appl. Math. Model.*, 2013, **37**(6), 4147–4164.
6. Gupta, A. K. and Khanna, A. Vibration of visco-elastic rectangular plate with linearly thickness variation in both directions. *J. Sound Vib.*, 2007, **301**(3–5), 450–457.
7. Khanna, A. and Arora, P. Theoretical analysis on thermally induced vibration of tapered parallelogram plate with mixed boundary conditions. *J. Vibroeng.*, 2014, **16**(3), 1276–1283.
8. Khanna, A. and Kaur, N. Effect of non-homogeneity on free vibration of visco-elastic rectangular plate with varying structural parameters. *J. Vibroeng.*, 2013, **15**(4), 2146–2155.
9. Khanna, A. and Kaur, N. Vibration of non-homogeneous plate subject to thermal gradient. *J. Low Freq. Noise V. A.*, 2014, **33**(1), 13–26.
10. Khanna, A. and Sharma, K. A. Natural vibration of visco-elastic plate of varying thickness with thermal effect. *Journal of Applied Science and Engineering*, 2013, **16(**2), 135–140.
11. Lal, R. and Kumar, Y. Characteristic orthogonal polynomials in the study of transverse vibrations of nonhomogeneous rectangular orthotropic plates of bilinearly varying thickness. *Meccanica*, 2012, **47**(1), 175–193.
12. Lal, R., Kumar, Y., and Gupta, U. S. Transverse vibrations of non-homogeneous rectangular plates of uniform thickness using boundary characteristic orthogonal polynomials. *Int. J. Appl. Math. Mech.*, 2010, **6**(14), 93–109.
13. Leissa, A. W. *Vibration of Plate*. NASA SP-160, U. S. Govt. Printing Office, 1969.
14. Patel, D. S., Pathan, S. S., and Bhoraniya, I. H. Infuence of stiffeners on the natural frequencies of rectangular plate with simply supported edges. *International Journal of Engineering Research & Technology*, 2012, **1**(3), 1–6.
15. Sharma, S., Gupta, U. S., and Lal, R. Effect of pasternak foundation on axisymmetric vibration of polar orthotropic annular plates of varying thickness. *J. Vib. Acoust.*, 2010, **132**(4), 1–13.
16. Tariverdilo, S., Shahmardani, M., Mirzapour, J., and Shabani, R. Asymmetric free vibration of circular plate in contact with incompressible fluid. *Appl. Math. Model.*, 2013, **37**(1–2), 228–239.
17. Wang, H. J. and Chen, W. L. Axisymmetric vibration and damping analysis of rotating annular plates with constrained damping layer treatments. *J. Sound Vib.*, 2004, **271**(1), 25–45.

Struktuursete parameetrite mõju ristkülikulise mittehomogeense viskoelastse plaadi vibratsioonidele

Anupam Khanna ja Narinder Kaur

On arendatud matemaatilist mudelit ristkülikulise mittehomogeense viskoelastse plaadi vibratsioonide kirjeldamiseks. On käsitletud termiliselt genereeritud vibratsioone neljast küljest jäigalt kinnitatud muutuva paksusega plaadis. Materjali mittehomogeensust arvestatakse kui eksponentsiaalset muutust Poissoni teguris ja temperatuur varieerub nii plaadi pikkuse kui ka laiuse sihis paraboolselt. On püstitatud rajaväärtusülesanne neljandat järku osatuletistega liikumisvõrrandile. On kasutatud Kelvini tüüpi viskoelastse materjali mudelit ja läbipainded on lineaarsele teooriale kohaselt väikesteks arvatud. Artiklis on vaadeldud termilise gradiendi, muutuva plaadi paksuse, plaadi pikkuse-laiuse suhte ja mittehomogeensuse konstandi mõju plaadi vibratsioonidele. Kasutades Rayleigh'i-Ritzi meetodit, on esitatud tulemused esimese kahe võnkevormi perioodi ja läbipainde jaoks. Tulemusi on võrreldud muude kirjanduses leiduvate andmetega.