Preface

The present issue of the *Proceedings of the Estonian Academy of Sciences* comprises selected articles written as an outcome of the Workshop on Interpolation, Inequalities, Invariants, and Related Topics, held in Lund University, Sweden, in October 2007. This was a workshop in a series of events which took place in Lund on the initiative of Professor Jaak Peetre. The first of them was way back in 1982, an intimate workshop on interpolation theory with some 10 participants. The 2007 workshop was organized with support from the Swedish Foundation for International Cooperation in Research and Higher Education STINT, the Crafoord Foundation, and the Royal Swedish Academy of Sciences. It was a follow-up to the Workshops on Trilinear Forms in Hilbert Space, Interpolation, and Related Topics held in Lund in 2003 and in 2005. This time it was decided to expand the scope by including some further topics related to interpolation, such as inequalities, invariant theory, symmetric spaces, operator algebras, multilinear algebra and division algebras, operator monotonicity and convexity, functional spaces and applications and connections of these topics to nonlinear partial differential equations, geometry, mathematical physics, and economics.

This issue is devoted to the above-mentioned directions and their interplay and applications, having interpolation, inequalities, operator and algebraic structures as the leitmotiv. The article of Jaak Peetre brings to light some intriguing connections between invariant theory, multilinear and especially trilinear forms, hyperdeterminants, bounded symmetric domains and harmonic analysis, actions of Lie groups and weighted Bergman spaces including several interesting directions for further investigation. The article by Amiran Gogatishvili, Alois Kufner, and Lars-Erik Persson discusses recent results, questions, and new methods on Hardy-type inequalities exhibiting the new scales of integral conditions, which yield, when applied to the original Hardy-type inequality situation, a new proof of a number of characterizations of the Hardy inequality and also some new weight characterizations. The article by Michael Cwikel is devoted to a more than 40 years open problem of whether an operator mapping one Banach couple boundedly into another and acting compactly on one (or even both) of the 'endpoint' spaces also acts compactly between the complex interpolation spaces generated by these couples. In the article this question is answered affirmatively in certain cases where the 'range' Banach couple is a couple of lattices on the same measure space. The article by Irina Asekritova and Natan Kruglyak provides a new algorithm for the construction of a near-minimizer for the couple of spaces based on the Besicovitch covering theorem and analysis of local approximations. In the article by Magnus Fontes and Olivier Verdier new results and applications of interpolation between anisotropic Sobolev spaces to partial differential equations, and the Burgers equation in particular, are presented. Interpolation has interesting deep connections to Pick functions, operator convex and operator monotone functions, and Loewner theory. In the article by Frank Hansen the remarkable new applications of operator monotone functions in economics are developed, involving two notions in microeconomics, decreasing relative risk premium and risk vulnerability. Extensions of operator convex functions to the context of operator algebras are considered and applied to convexity characterizations of commutative and of subhomogeneous C^*-algebras in the article by Sergei Silvestrov, Hiroyuki Osaka, and Jun Tomiyama.

Finally, the article by Erik Darpö presents interesting new aspects and results pertaining to the problem of classification of finite-dimensional real division algebras with deep connections to geometry and invariants.

Following success of the previous workshops, which generated substantial research advances and important publications and cooperation, we plan new similar workshops in 2010 and 2012.

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Guest editors of the issue:
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