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ON EFFICIENCY OF OPTIMIZATION IN POWER SYSTEMS

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A method and the main results of the study carried out for the evaluation of optimization efficiency in power systems are presented in the paper. Two classic optimization problems were studied: 1) economic dispatch problems of thermal power units, 2) unit commitment problems. In both tasks the total fuel costs and the environmental impacts were minimized. The study showed that the maximal efficiency of optimization in thermal power plants and in power systems may reach 30% and even more.

Introduction

The basic function of a power system is to supply its customers with electrical energy at the competitive unit price as economically as possible. Therefore the reliability of supply and the quality of electrical energy must correspond to the relevant standards and norms. Power systems must be possibly sustainable, secure and safe for environment.

Operation and development of the power systems must be optimized [1–8]. This enables to minimize the fuel cost, the cost of operation, the expected investments and/or the environmental impacts. Nowadays the theory and methods for optimization of power systems enable to take into account different types of constraints and to consider not only the deterministic information, but probabilistic, uncertain or fuzzy information as well [8]. However, the interest of energy companies to optimization has been relatively small yet. Probably they hope that the competition and electricity markets will fill the place of optimization, but this is not so. The optimization of electricity generation, transmission and distribution is also needed in the energy market conditions.

In scientific literature there is relatively small information about practical efficiency of optimization in power plants and systems. In this paper a

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method and the main results of the study of the efficiency of optimization in power systems are presented. Two classic optimization problems have been studied: 1) economic load dispatch problems of thermal power units, 2) unit commitment problems.

Efficiency of economic load dispatch problem

Method

In general the economic dispatch problem of operating thermal units is a vector optimization problem. For example, the minimization of the total fuel costs and the environmental impacts is very essential. Ordinarily this problem is taken as a mono optimization problem.

We started to solve the problem

$$\text{minimizing } B = \sum_{i=1}^{n} B_i(P_i) . \quad (1)$$

Subject to the following constraints:

The active power balance

$$P_D + \sum_{i=1}^{n} P_i^{Aux} + P_L - \sum_{i=1}^{n} P_i = 0 , \quad (2)$$

The active power limits to the power units

$$P_i^- \le P_i \le P_i^+ ,\ i = 1, \dots , n, \quad (3)$$

where

- B – total fuel cost of thermal power units,
- $B_i(P_i)$ – fuel cost characteristic of unit *i*,
- P_D – total power demand from thermal units,
- $P_i^{Aux}(P_i)$ – auxiliary power characteristic of unit *i*,
- P_L – total active power losses,
- P_i – active load of unit *i*,
- P_i^-, P_i^+ – minimum and maximum limits on the generator outputs,
- n – number of operating units.

Here $P_1,...,P_n$ are the controllable variables, P_D is a non-controllable variable, and fuel cost characteristics, auxiliary characteristics and active power losses are non-controllable functions.

The Lagrange function for the problem (1)–(2) without the inequality constraints (3) is as follows:

$$\Phi = \sum_{i=1}^{n} B_i(P_i) + \mu(P_D + \sum_{i=1}^{n} P_i^{Aux} + P_L - \sum_{i=1}^{n} P_i) , \quad (4)$$

where μ – Lagrange multiplier (controllable variable).

Now let us take the partial derivatives from the Lagrange function (4) with respect to each of the controllable variables and equal these to zero:

$$\frac{\partial \Phi}{\partial P_i} = \frac{\partial B_i}{\partial P_i} + \mu\left(\frac{\partial P_i^{Aux}}{\partial P_i} + \frac{\partial P_L}{\partial P_i} - 1\right) = 0\,, \tag{5}$$

$$\frac{\partial \Phi}{\partial \mu} = P_D + \sum_{i=1}^{n} P_i^{Aux} + P_L - \sum_{i=1}^{n} P_i = 0\,. \tag{6}$$

The equations (5) and (6) are the necessary optimality conditions for solving problem (1), (2) without the inequality constraints (3).

The conditions (5) can be written in the following form:

$$\frac{\dfrac{\partial B_i}{\partial P_i}}{1 - \dfrac{\partial P_i^{Aux}}{\partial P_i} - \dfrac{\partial P_L}{\partial P_i}} = \mu\,, \qquad i = 1, \dots, n \tag{7}$$

where

$b_i = \dfrac{\partial B_i}{\partial P_i}$ – incremental fuel cost with respect to unit cross load P_i,

$\alpha_i = \dfrac{\partial P_i^{Aux}}{\partial P_i}$ – incremental auxiliary power with respect to unit cross load,

$\sigma_i = \dfrac{\partial P_L}{\partial P_i}$ – incremental losses with respect to unit cross load.

Similarly the conditions of optimality for minimization of environmental impacts can be derived. In order to that, the fuel cost characteristics must be replaced with corresponding environmental impact characteristics (with CO_2 characteristics, SO_2 characteristics or NO_X characteristics).

The efficiency of optimization $\Delta B^{\%}$ is calculated by the following formula:

$$\Delta B^{\%} = \frac{B(P) - B(P^0)}{B(P^0) - B(P^-)} \cdot 100\%\,, \tag{8}$$

where

$B(P^0) = \min B$ – total fuel cost at optimal load distribution $P^0 =< P_1^0, ..., P_n^0 >$,

$B(P)$ – total fuel cost at non-optimal load distribution $P = < P_1, ..., P_n >$,

$B(P^-)$ – fuel cost at minimum loads of units $P^- = < P_1^-, ..., P_n^- >$.

The efficiency of optimization on the occasion of tasks (1)–(3) shows how many the optimal load distribution P^0 can decrease the value of objective function in percents compared with the load distribution P. The main interest offers the maximum value of efficiency:

$$\Delta B^{\%}_{Max} = \frac{B(P^{00}) - B(P^{0})}{B(P^{0}) - B(P^{-})} \cdot 100\%, \tag{9}$$

where $B(P^{00}) = \max B$ – total fuel cost at the maximizing load distribution $P^{00} = < P_i^{00},..., P_n^{00} >$.

The factual efficiency of optimization depends on a lot of factors and will be in the interval:

$$0 \le \Delta B^{\%} \le \Delta B^{\%}_{Max} . \tag{10}$$

General principles for the determination of maximum efficiency of the optimization are as follows: 1) finding the load distribution that minimizes the objective function, 2) finding the load distribution that maximizes the objective function, 3) finding maximum efficiency of optimization in percents.

The efficiency of optimization was studied from the theoretical and experimental aspects. The experimental study occurred by the special programs composed for this project.

The characteristics used in this project were chosen based on characteristics presented in [3, 4, 8]. They were widely modified. The main attention for evaluation of $\Delta B^{\%}_{Max}$ offered the following two types of input-output characteristics: 1) The $B_i(P_i)$ characteristics of units are continuous, monotonically increasing, convex and smooth functions; 2) The $B_i(P_i)$ characteristics of units are continuous, increasing and convex functions, but they have a broken point. The typical fuel cost and incremental fuel cost characteristics are shown in Fig. 1–2.

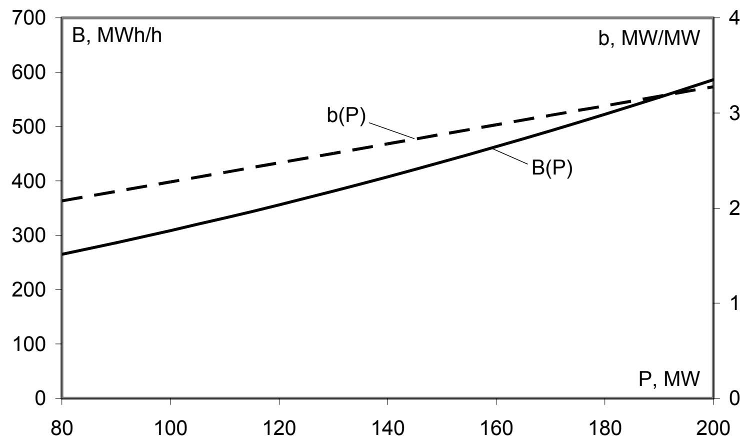

Fig. 1. Typical smooth fuel cost and continued incremental fuel cost characteristics of thermal power unit.

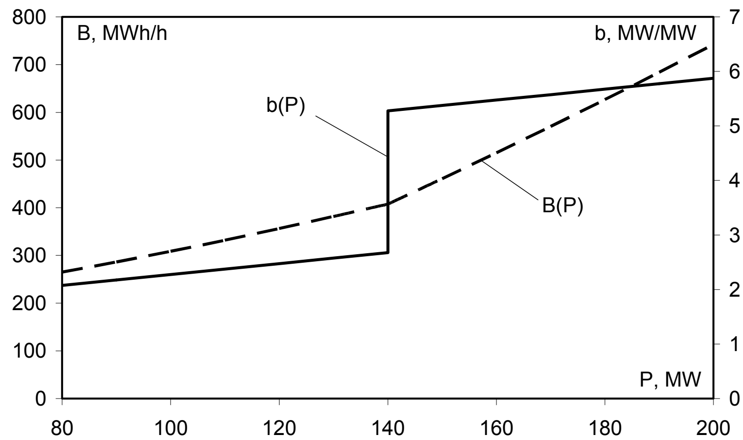

Fig. 2. Typical fuel cost characteristic with broken point and incremental fuel cost characteristic of thermal power unit.

Main results of analysis

1. The efficiency of the optimization in the case of economic load dispatch problem depends mainly on the controllability of load distribution and on the characteristics of power units. The controllability of load distribution in turn depends on the value of power demand P_D and on the limits of generator outputs. The influence of auxiliary power and consideration of power losses are noticeably less. Therefore for evaluation of the maximum efficiency different simplified models and even the models with two power units can be used. A typical function of $\Delta B^{\%} = \Delta B^{\%}(P_D)$ is shown in Fig. 3.

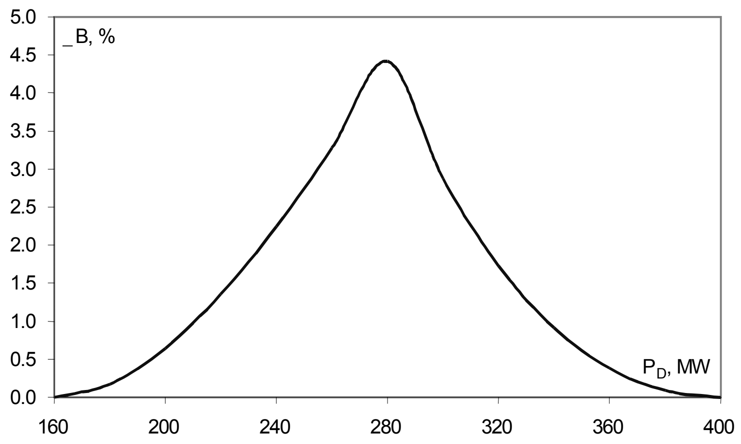

Fig. 3. Typical form of $\Delta B^{\%}(P_D)$.

2. In the case of two power units model with the smooth and identical fuel cost characteristics, the efficiency of economic load dispatch can be calculated by the following simplified formula:

$$\Delta B = b_i^{'} \Delta P_i^2 , \tag{10}$$

where

$b_i^{'} = \dfrac{\partial b_i}{\partial P_i} = \dfrac{\partial^2 B_i}{\partial P_i^2}$ – the first derivative of $b_i(P_i)$ or the second derivative of $B_i(P_i)$,

ΔP_i – deviation of unit's load from the optimal value.

Typical functions of $\Delta B^{\%}(b_i^{'})$ and $\Delta B^{\%}(\Delta P_i)$ are shown in Fig. 4 and 5.

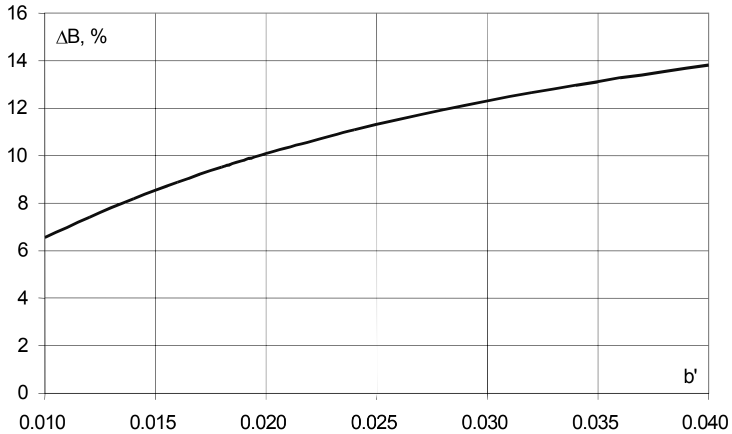

Fig. 4. Typical function of $\Delta B^{\%}(b_i^{'})$.

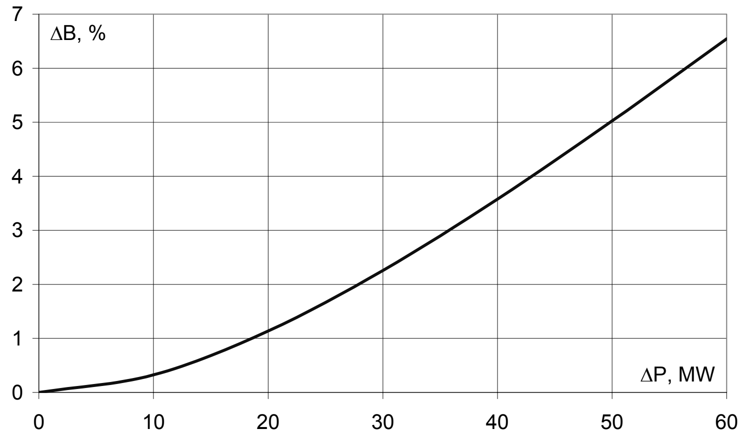

Fig. 5. Typical form of $\Delta B^{\%}(\Delta P_i)$.

3. The calculations carried out for the different characteristics of power units, for the different values of load demand and for the different limits of units load show that the maximum efficiency of optimization for optimal load dispatch problem of the thermal units is changing and may be very high. It may reach 30% and even more due to the optimal fuel cost component depending of unit's load. The average value of the efficiency for the optimal load dispatch problems is approximately 10–15%.
4. The maximal efficiency of minimization the environmental impacts are in some measure smaller, about 20%. The average value of efficiency for minimization of environmental impacts is about 10%.

Efficiency of unit commitment problem

Method

Unit commitment is the problem of determining the schedule of generating units. The solution of this problem will determine the objects that must be on or off. The objective functions to be minimized are the fuel costs, the maintenance costs and the start up costs [6]. The most known methods for the solution of the unit commitment problem are [4]:

1) Priority-list methods,
2) Dynamic programming method,
3) Lagrange relaxation method.

The important indicators for start-up and shut-down of units are their fuel cost rates $\delta_i = B_i / P_i$ and the fuel cost rate characteristics $\delta_i(P_i)$. At that many factors are taken into consideration as unit operating constraints and costs, generation and reserves constraints, unit start-up constraints and others [4, 6–8].

The efficiency of the unit commitment problem was analyzed by using special programs [8].

The maximum efficiency of the unit commitment optimization was determined by the following formula:

$$\Delta B_{Max}^{\%} = \frac{MaxB - MinB}{MinB} \cdot 100\% , \tag{11}$$

where $MinB$ – minimum of total fuel cost.

Main results of the analysis

1. The efficiency of the optimal unit commitment depends on several factors. The calculation shows that the maximum efficiency of the unit commitment optimization only is in some measures smaller than the efficiency of load distribution optimization and forms approximately 5–10% of the total fuel cost. The expected efficiency of the unit commitment optimization is about 3–6%.

2. A typical dependence of fuel cost from the number of operating units is shown in Fig. 6.

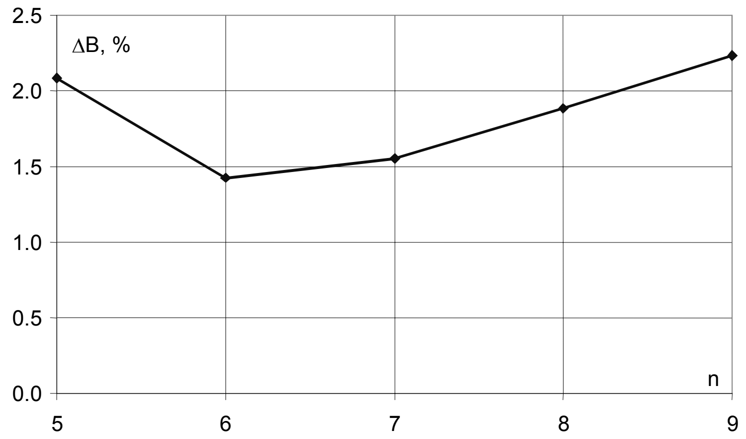

Fig. 6. Typical form of $\Delta B^{\%}(n)$.

Conclusions

1. This study shows that the optimization in power systems is of very great importance. The maximum efficiency of optimization only for load distribution and unit commitment problems in thermal power plants and power systems can decrease the fuel cost and economical impacts of thermal units approximately by 10–20%.
2. The optimization is the cheapest possibility of economizing on energy resources, thermal power plants and power systems, of minimizing the operation and investment costs and of reducing environmental impacts.
3. The methods of optimal control and optimal planning of power systems' development can find wide application in the energy market conditions.

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