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TYPE MODELS OF ELECTRICAL NETWORK LOAD

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The mathematical model of load, used in the monitoring of electrical network operational dynamics, is suitable to describe different types of loads. The structure of the mathematical model is the same for all loads. In order to use the load model to describe specific loads, the model parameters must be estimated. If the existing data is not sufficient to evaluate all parameters of the model, type-models – a typical sets of model parameters – may be used. Irrespective of initial information, the result is always a complete model that describes all necessary details of the load. However, the accuracy of the model and its applications are dependent on availability of initial data and on the quality of type models.

Introduction

Mathematical model [1], describing timely changes, stochasticity and dependency on weather and state variables of electrical network load, may be applied to describe different types of loads. It is possible to observe total active and reactive loads of the whole power system or of some region, e.g. distribution network, different bus loads, but also loads of separate consumers. The scale of power may reach from some gigawatts to some hundred watts. It is understandable that the amount and quality of initial information is different. In the case of large loads, time series, where active and reactive power values are fixed hourly or more frequently, are available for many years. On the other hand, in the case of private consumers or planned industry consumers it is possible that only one number, yearly energy demand, may be available.

The mathematical model includes a large number (1000) of parameters. The modelling principle is that in any case all those parameters must be assessed for each load. Simplified models caused by lack of load data are not observed. If initial data does not allow to estimate all model parameters, only a part of parameters is directly obtained basing on available data. Remaining parameters are transferred from a type model, i.e. from some previously

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estimated load model, which in its nature corresponds to the observable load. If there is enough data available, e.g. hourly data at least for some years, and they are of necessary quality, then it is possible to estimate a unique model for every load. No computational obstacles thereby exist. However, type models are often needed. The main reason is insufficiency of initial data due to unavailability of data or a remarkable change in the nature of the load whereby earlier data is not applicable. The necessity to implement type models may rise even when enough load data is available. For example, in a distribution network where the stochasticity of load is high, type models may enhance the reliability of model parameters. When modelling the distribution network operation it is necessary that some model parameters, which form the model co-ordinates, are the same for all loads [2, 3].

The hierarchy of model parameters contributes the use of type models. Actually, some model parameters are relatively unchangeable in time. Other parameters are more dependent on the change of the loads nature. The parameters belonging to the first group are determined during the initial estimation of load models, i.e. during load research. Those parameters may principally be classified and used as type models, i.e. if necessary they may be used in other load models. The second group consists of parameters that are more adapted to a certain load and ought to be estimated according to the specific load data. Those parameters should be adjusted separately for every load. Estimation of parameters of the second group is also necessary when the nature of the load changes with time.

Principally, whatever load model, parameters of which are known, may be used as a type model. However, transfer of parameters form a type model may not happen randomly. It is necessary that the nature of the loads are somewhat similar. The concept of similarity here is different from the traditional concept. The similarity of the shape of the load curve and the level of load (trend) may not be essential, because those properties may be considered even with a small number of parameters, which are estimated separately according to each load. In this paper, a possibility of using the main parameters of the same model, i.e. model co-ordinates, for different loads is observed. New loads, for which earlier models are absent and available load data is insufficient, and the above mentioned modelling of distribution network operation are considered as applications.

Mathematical model of load

The mathematical model describing load changes (active power, reactive power, or current) consists of three basic components:

$$P(t) = E(t) + \Gamma(t) + \Theta(t) \, ,$$

where $E(t)$ is the mathematical expectation of the load,
$\Gamma(t)$ is the temperature-sensitive part of the load,
$\Theta(t)$ is the stochastic component of the load.

Mathematical expectation describes regular changes of a load, such as general trend and seasonal, weekly, and daily periodicity. Mathematical expectation is principally non-stochastic and corresponds to the normal temperature.

The temperature-sensitive part of a load describes load drift, caused by deviations of outdoor temperature from the normal temperature. The normal temperature (mathematical expectation of the temperature) is the average outdoor temperature of the last 30 years on any given hour of the year. Besides other features, temperature dependency of loads is characterized by a delay of about 24 hours. If the actual outdoor temperature corresponds to the normal temperature (considering delay), the influence of temperature is lacking. In order to compare the temperature dependencies of different loads, component $\Gamma(t)$ is normalized

$$\Gamma(t) = R(t)\gamma(t),$$

where $R(t)$ is the rate of the temperature dependency of the load,

$\gamma(t)$ is the normalized temperature dependency component.

The stochastic component describes stochastic deviations of load. Due to the autocorrelation, the deviations of load are stochastically dependent on each other. It is possible to observe the stochastic component of the load as consisting of expected deviation $\zeta(t)$, which describes the conditional mathematical expectation of the stochastic component and normally distributed non-correlated residual deviation (white noise) $\xi(t)$. In addition, it is necessary to observe peak deviations of the load by the component $\pi(t)$, describing large positive or negative deviations that do not correspond to the normal distribution. It is also rational to normalize the stochastic component. The proper rate here is standard deviation of the load $S(t)$. The result is

$$\Theta(t) = S(t)\left[\zeta(t) + \xi(t) + \pi(t)\right].$$

Hence, mathematical model of load takes the following form:

$$P(t,h,l) = E(t,h,l) + R(t,h,l)\gamma(t) + S(t,h,l)\left[\zeta(t) + \xi(t) + \pi(t)\right],$$

where load mathematical expectation, standard deviation, and rate of temperature dependency are observed as a function of yearly (general) time t, daily time h, and type of day l.

Mathematical expectation, standard deviation, and rate of temperature dependency can be described by the following expressions:

$$E(t,h,l) = \mathbf{M}^T(h)\mathbf{G}_{El}\mathbf{N}(t),$$

$$S(t,h,l) = \mathbf{M}^T(h)\mathbf{G}_{Sl}\mathbf{N}(t),$$

$$R(t,h,l) = \mathbf{M}^T(h)\mathbf{G}_{Rl}\mathbf{N}(t),$$

where $\mathbf{M}(h)$ and $\mathbf{N}(t)$ are vector functions which include components corresponding to daily and annual load changes.

$$\mathbf{M}(h) = \begin{bmatrix} \mu_0(h) \\ \mu_1(h) \\ \dots \\ \mu_{MDC}(h) \end{bmatrix}, \qquad \mathbf{N}(t) = \begin{bmatrix} \nu_0(t) \\ \nu_1(t) \\ \dots \\ \nu_{NAC}(t) \end{bmatrix}.$$

Matrices $\mathbf{G}_{El}$, $\mathbf{G}_{Sl}$, and $\mathbf{G}_{Rl}$ consist of parameters depending on day type l

$$\mathbf{G}_{El} = \|g_{Elks}\|, \ \mathbf{G}_{Sl} = \|g_{Slks}\|, \ \mathbf{G}_{Rl} = \|g_{Rlks}\|,$$

where $k = 0 - MDC$ and $s = 0 - NAC$. Here the vector components, corresponding to index 0, are trivial

$$\mu_0(h) \equiv 1, \ \nu_0(t) \equiv 1.$$

The number of non-trivial components MDC and NAC is, for example, 4–5. In Figs 1 and 2, some examples of the vector function components, $\mathbf{M}(h)$ and $\mathbf{N}(t)$, are shown.

Matrices $\mathbf{G}_\mathbf{E}(l)$, $\mathbf{G}_S(l)$, and $\mathbf{G}_R(l)$ can be developed into series

$$\mathbf{G}_{El} \cong a_{l0}\mathbf{G}_0 + a_{l1}\mathbf{G}_1 + a_{l2}\mathbf{G}_2 + \ldots + a_{l,NSC}\mathbf{G}_{NSC},$$

$$\mathbf{G}_{Sl} \cong b_{l0}\mathbf{G}_0 + b_{l1}\mathbf{G}_1 + b_{l2}\mathbf{G}_2 + \ldots + b_{l,NSC}\mathbf{G}_{NSC},$$

$$\mathbf{G}_{Rl} \cong c_{l0}\mathbf{G}_0 + c_{l1}\mathbf{G}_1 + c_{l2}\mathbf{G}_2 + \ldots + c_{l,NSC}\mathbf{G}_{NSC},$$

where $\mathbf{G}_0 = \|1\|$.

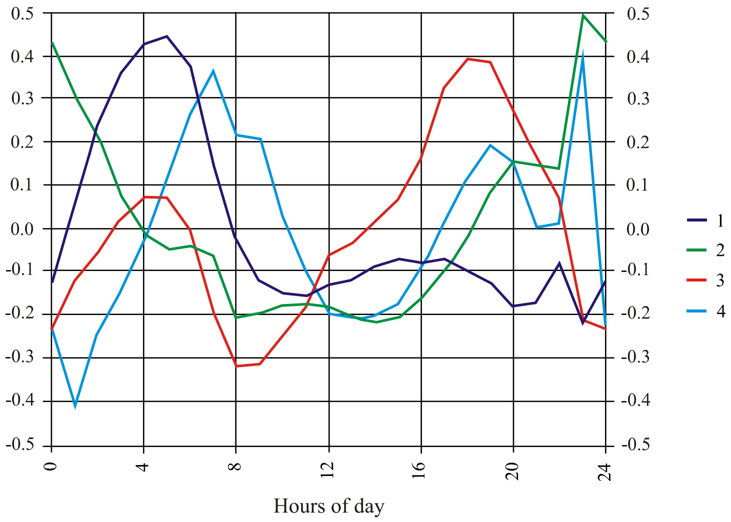

Fig. 1. Components of vector function $\mathbf{M}(h)$.

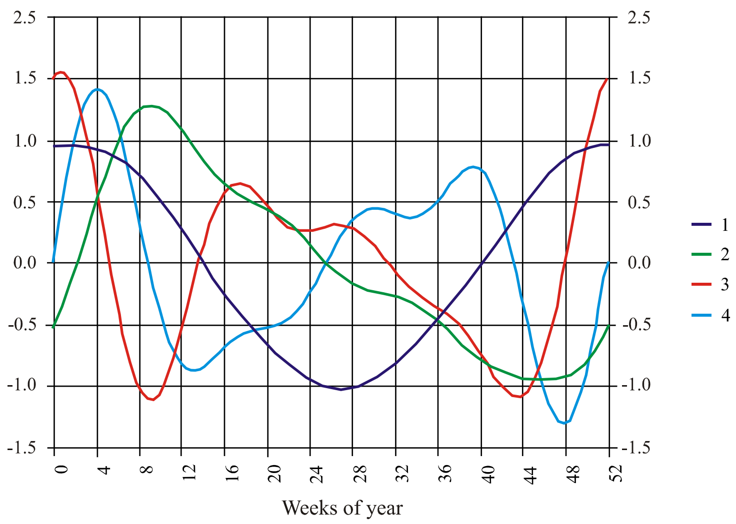

Fig. 2. Components of vector function $\mathbf{N}(t)$.

The result is

$$E(t,h,l) = \mathbf{M}^T(h)\sum_r (a_{lr}\mathbf{G}_r)\mathbf{N}(t),$$

$$S(t,h,l) = \mathbf{M}^T(h)\sum_r (b_{lr}\mathbf{G}_r)\mathbf{N}(t),$$

$$R(t,h,l) = \mathbf{M}^T(h)\sum_r (c_{lr}\mathbf{G}_r)\mathbf{N}(t),$$

where r = 0–NSC. Actually, NSC = 10–12.

Types of day l = 1–NTP correspond, first of all, to normal weekdays (l = 1–7). In addition, special days (holidays, pre-holidays, post-holidays, etc.), for which $l > 7$, are observed. The number of special days depends on the calendar (the country) requiring accuracy of modelling. The total number of type of days NTP may be up to 50–60. In a simplified case, the special day is considered as a similar weekday (holiday – Sunday, pre-holiday – Friday, etc). In that case, the number of different types of days is 7.

Model parameters a_{lr}, b_{lr}, and c_{lr} can be normalized, based on mean value of mathematical expectation, standard deviation, and rate of load model, respectively.

$$a = \frac{1}{7}\sum_{l,r} a_{lr} g_{r00}, \quad b = \frac{1}{7}\sum_{l,r} b_{lr} g_{r00}, \quad c = \frac{1}{7}\sum_{l,r} c_{lr} g_{r00},$$

where g_{r00} is the element of matrix $\mathbf{G}_r$ with index 00. Here the summing up is done in the range of ordinary weekdays l = 1–7 (special days are not considered). In the model, elements of matrices are replaced as follows:

$$a_{lr} \Rightarrow a \cdot a_{lr}\,,\, b_{lr} \Rightarrow b \cdot b_{lr}\,,\, c_{lr} \Rightarrow c \cdot c_{lr}\,.$$

Parameters cannot be normalized if the calculated mean value is too small (zero) or negative, which may occur in the case of reactive power and current. In that case

$$a = b = c = 1.$$

The load level and shape may change. Fast changes may be considered in the model as a step change of the model factors at a certain time. Long-term load increase or decrease is presented as an additional component of the model – trend. Trend component is given as quadratic function:

$$A(t) = a\left[1 + \alpha_1(t - t_0) + \alpha_2(t - t_0)^2\right] \;,$$

where α_1 and α_2 are factors, and t_0 is the moment in time at which the computation of trend starts. Beside mathematical expectation, trend also belongs to the standard deviation and the rate of the temperature dependency as follows:

$$B(t) = b\left[1 + \alpha_1(t - t_0) + \alpha_2(t - t_0)^2\right] \;,$$

$$C(t) = c\left[1 + \alpha_1(t - t_0) + \alpha_2(t - t_0)^2\right] \;,$$

where factors α_1 and α_2 as well as t_0 are for practical purposes the same as in the case of mathematical expectation.

Load voltage and frequency sensitivity are described as quadratic functions:

$$U(u) = 1 + \mu_1 u + \mu_2 u^2\,,\ \text{ where }\ u = U_V / U_N - 1\,,$$

$$F(f) = 1 + \nu_1 f + \nu_2 f^2\,,\ \text{ where }\ f = F_V / F_N - 1\,.$$

Here $\mu_1, \mu_2, \nu_1, \nu_2$ are factors and U_V, U_N and F_V, F_N are rated values of voltage and frequency, respectively. Dependency on voltage and frequency is considered only in connection with mathematical expectation. Therefore

$$E(t,h,l) = A(t)U(u)F(f)\mathbf{M}^T(h)\sum_r\left(a_{lr}\mathbf{G}_r\right)\mathbf{N}(t)\,,$$

$$S(t,h,l) = B(t)\mathbf{M}^T(h)\sum_r\left(b_{lr}\mathbf{G}_r\right)\mathbf{N}(t)\,,$$

$$R(t,h,l) = C(t)\mathbf{M}^T(h)\sum_r\left(c_{lr}\mathbf{G}_r\right)\mathbf{N}(t)\,.$$

Model co-ordinates and factors

Model parameters may be divided into two groups. The first group, named model co-ordinates, include vector functions $\mathbf{M}(h)$ and $\mathbf{N}(t)$ and matrices $\mathbf{G}_r$. The second group, named model factors, include parameters a_{lr}, b_{lr}, c_{lr} (shape factors), a, b, c and other factors related to trend (level factors). The reason of the above described grouping is due to the fact that the level of load and shape of the load curve is first of all determined by model factors even if the model co-ordinates do not change. From this the idea accrues that one possibility to specify type models is transferring model co-ordinates from one model to another. It is also possible to estimate model co-ordinates mutually using multiple load data, whereas model factors are found separately for every specific load.

Besides model co-ordinates, sub model components of the load temperature dependency and stochasticity, $\gamma(t)$, $\zeta(t)$, $\xi(t)$ and $\pi(t)$, may be observed as typical. The mathematical representation of those components is not observed in this paper.

To investigate the composition and field of application of type models we observe 18 loads of 5–240 MW for 2–12 years. At first, the models are estimated uniquely, separately for each load data. Afterwards, based on all load data, the model mutual co-ordinates are estimated and model factors, based on specific load data for each load, are found.

The results of the estimation of mathematical expectation are summarized in Table 1. Here *P* describes average value of load and *S* is the average value of standard deviation. Deviations are found between the load values, normalized regarding the temperature and, mathematical expectation $\Delta P(t) = P(t) - \Gamma(t) - E(t)$. Values *dP* and *dS* indicate the increment of standard deviation ΔS due to the use of mutual co-ordinates – $dP = \Delta S / P$ and $dS = \Delta S / S$. From Table 1 it appears that the error, due to the transition of the type co-ordinates, is mostly below 1% at load average values and in the order of 10% at load standard deviation. These errors should be considered as acceptable.

Table 1. **Estimation errors of mathematical expectation**

Number	1	2	3	4	5	6	7	8	9
P, MW	4.5	4.9	8.6	10.8	14.6	18.0	18.3	18.4	29.1
S, MW	0.3	0.4	0.6	0.8	1.0	1.2	1.0	1.0	1.9
dP, %	0.6	1.1	0.6	0.6	0.6	0.4	0.5	0.6	1.3
dS, %	8.1	14.2	8.9	9.0	9.3	5.6	8.4	12.0	20.5
Number	10	11	12	13	14	15	16	17	18
P, MW	47.0	64.7	113.4	121.4	130.0	158.7	185.7	195.0	240.0
S, MW	3.0	3.5	6.3	4.8	8.6	7.1	8.3	9.4	12.6
dP, %	0.2	0.5	0.5	0.6	0.6	0.5	0.8	0.5	0.5
dS, %	3.0	9.6	8.9	14.7	9.8	12.7	18.2	9.8	9.4

Beside the average values of errors, it is interesting to see to what extent it is possible to represent different curves (mathematical expectation) of the load, using the type co-ordinates. In Fig. 3, Fig. 4 and Fig. 5 examples of normalized values of three loads and corresponding mathematical expectation values are presented. It is evident that in the same co-ordinates it is possible to represent different shapes of mathematical expectation curves.

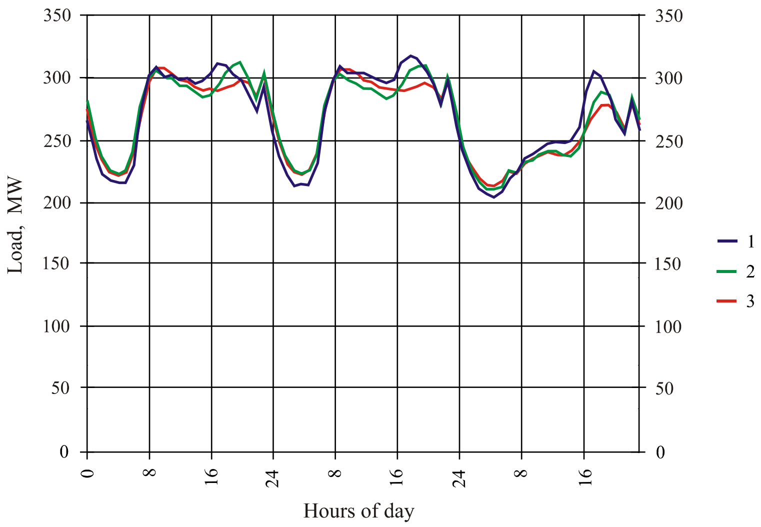

Fig. 3. Normalized value of load number 18 (1), and mathematical expectation based on unique model (2) and type model (3).

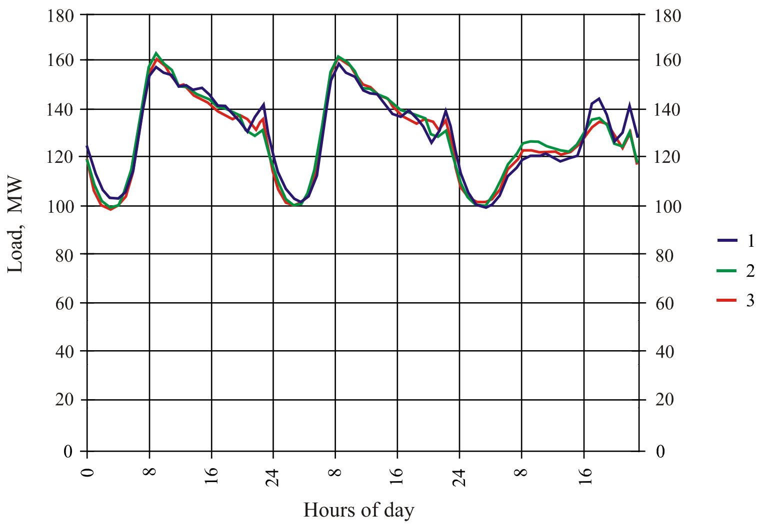

Fig. 4. Normalized value of load number 15 (1), and mathematical expectation based on unique model (2) and type model (3).

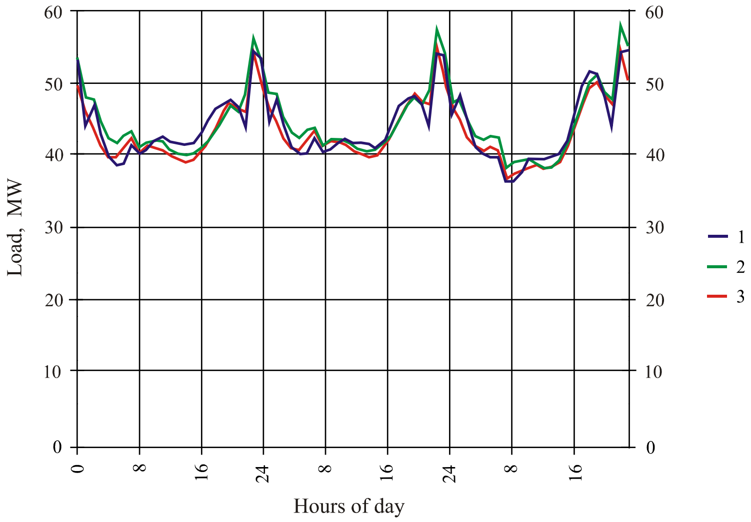

Fig. 5. Normalized value of load number 10 (1), and mathematical expectation based on unique model (2) and type model (3).

When observing a longer period of changes of load, e.g. a year, it is possible to notice that the values of mathematical expectation, based on a unique model and the values, based on a type model are practically identical. It should be emphasised that the mathematical expectation is found based on previously described relations, and model factors are found as averages up to the whole observable data interval (2–12 years).

The actual values of load are presented here only for comparison. Of course, when using the model for load forecasting, the adjustment of model factors (for example, once a year) is possible. During the forecasting, the actual values of load are used only for calculating deviation $\zeta(t)$, which, if added to the mathematical expectation in the form of $E(t)+\Gamma(t)+S(t)\zeta(t)$, gives the short-term forecast of the load.

In addition to mathematical expectation, also temperature dependency and standard deviation of load are of interest. The importance of temperature dependency may be high in some cases. In Fig. 6 an example of temperature influence on the load (number 18 in Table 1) in the winter period is presented. The average values of temperature influence vary in an interval between –60–+90 MW, which forms 62% of the average value of a load. Daily average values of temperature influence, which are calculated based on a unique model and a type model, practically coincide here.

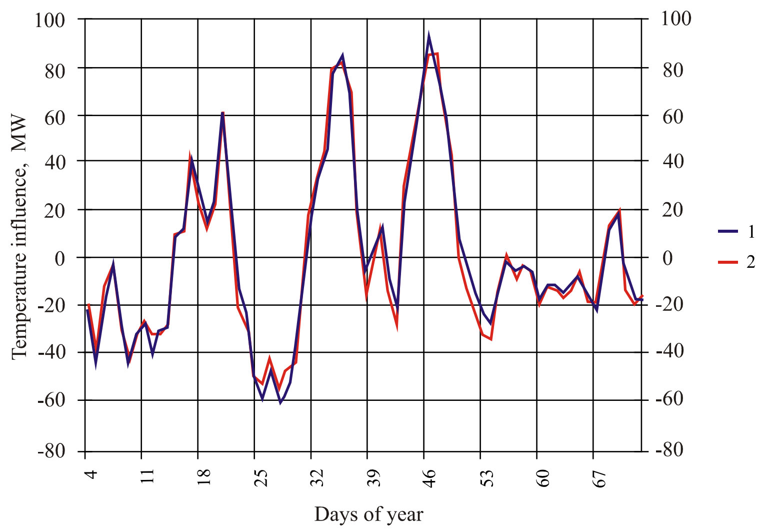

Fig. 6. Daily average values of temperature influence based on a unique (1) and a type model (2).

Conclusions

In the case of insufficient initial data or for some other reason, necessity for implementation of type models may arise when estimating the mathematical model of load. One version of type models is to identify model co-ordinates for different loads, i.e. representing the model in the same co-ordinates. Thereby, the model factors are found separately, based on specific load data. Based on the observed 18 loads, it is possible to indicate that such definition and implementation of the type model gives acceptable results.

However, it is not possible to generalize the obtained results for any loads. For every specific case the load research should be performed in order to determine which loads can and which cannot be modelled in the same co-ordinates. Nevertheless, it could be declared that the accepted co-ordinates may be used for a relatively long period of time (decades), while adjusting model factors only from time to time. Unique models, which are estimated separately according to a specific load, give more accurate results to a certain extent and are therefore preferable, if enough high quality load data is available.

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