Proc. Estonian Acad. Sci. Eng., 2007, **13**, 2, 84–93

On the uncertainty of measurements by measuring the form of a surface

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Received 2 November 2006, in revised form 29 January 2007

Abstract. Modern manufacturing poses high demands on the accuracy of surfaces of parts of machinery of complicated form. Whether the produced parts of machinery meet the demands, listed in technical specifications, can only be determined in the course of corresponding measurements. One of the possibilities of measuring the surface contour of the parts of complicated form is by using the inductive surface roughness and form measurement instrument "Perthometer Concept". Although this measuring instrument has been previously calibrated, an investigation was carried out to assess the reliability of the measurement results. A measurement model was composed and the values of the input quantities as well as their distribution were experimentally determined. As a result of this investigation, the reliability of the surface measurement results are characterized basing on expanded uncertainty.

Key words: surface form, measurement instrument, uncertainty of measurement.

1. INTRODUCTION

Uncertainty of measurement is, by its definition, a parameter, associated with the result of the measurement and characterizing the dispersion of the values that can be attributed to the measurand [1,2]. It reflects the lack of exact knowledge about the value of the measurand. Thus owing to the uncertainty, arising from random effects and from imperfect correction of the result for systematic effects, the result of the measurement after correction for recognized systematic effects is still only a rough estimate of the true value of the measurand. For this reason, each measuring result should be associated with information about the uncertainty, identifying the possible dispersion of the true value of the measurand. In metrology laboratories, mostly standardized procedures are used in evaluating measuring uncertainty. However, these procedures require extended statistical and mathematical knowledge, which usually is not available in industry.

In [3–6], surface roughness was measured with a roughness measuring instrument. The uncertainty of measurement results could be estimated only by the uncertainty contribution of the measuring instrument. This forms about 10–15% of the total indication. Besides that, no other contributors of uncertainty were considered to estimate the measurement results. In [7–9], step height was measured with a surface roughness measuring instrument. To evaluate the measurement results, in addition to the uncertainty, contributed by the measuring instrument, the uncertainty caused by the measurer was considered. As a result, the measurement results became more reliable. In above-mentioned papers, however, other contributions to uncertainty – the stylus radius, measurement force, surface angle, etc. – were ignored.

The current research has the aim to consider all the possible uncertainty contributors, essential in estimating a surface contour of a complicated form.

2. EVALUATION OF THE UNCERTAINTY OF MEASUREMENTS

Uncertainty in measurement comprises, in general, many components. Some of these components may be evaluated from the statistical distribution of the results of a series of measurements and can be characterized by experimental standard deviations (type A evaluation of uncertainty). The other components, which can also be characterized by standard deviations, are evaluated from assumed probability distributions, based on experience or other information (type B evaluation of uncertainty).

According to the reference document [1], the first step in determining the uncertainty of a measurement is to calculate the model function f that shows the relationship between the input quantities $(X_1, X_2, ..., X_N)$ and the quantity to be measured Y:

$$Y = f(X_1, X_2, ..., X_i, ..., X_N). \quad (1)$$

Model function f represents the procedure of measurement and the method of evaluation. It describes how values of the output quantity Y are obtained from the values of the input quantities X_i. In most cases it is an analytical expression, but it may also be a group of such expressions which include corrections and correction factors for systematic effects, thereby leading to a more complicated relationship that cannot be written down as an explicit function. In this case, f may be determined numerically.

An estimate of the measurand Y, the output estimate denoted by y, is obtained from Eq. (1) using input estimates x_i for the values of the input quantities X_i:

$$y = f(x_1, x_2, ..., x_i, ..., x_N). \quad (2)$$

First, standard uncertainties $u(x_i)$ of all input estimates x_i should be evaluated. For an input estimate of X_i, obtained from the statistical analysis of a series of observations (type A evaluation of uncertainty), standard deviation of the mean value of X_i is calculated as

$$s(\overline{x}_i) = \frac{s(x_{i,j})}{\sqrt{n_i}}. \tag{3}$$

Variance $s^2(x_{i,j})$ for non-correlated input values is calculated as

$$s^2(x_{i,j}) = \frac{1}{n_i - 1}\sum_{i=1}^{n_i}(x_{i,j} - \overline{x}_i)^2. \tag{4}$$

If some of the input quantities are correlated, the correlation should be considered in Eq. (4).

Standard uncertainty $u(x_i)$ is equal to the standard deviation of the mean

$$u(x_i) = s(\overline{x}_i). \tag{5}$$

Input values are the best estimates that were corrected in terms of all effects, significant for the model. If that was not the case, the necessary corrections were introduced as separate input quantities.

Due to insufficient knowledge, estimations of the input quantities are not exact, leading to uncertainty, characterized by standard deviation of the output quantity *Y*. The calculation of the output quantity is performed applying the law of propagation of variances to Eq. (1):

$$u(y) = \sqrt{\sum_{i=1}^{N}\left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i)}. \tag{6}$$

Therefore it is necessary to know standard deviations, called standard uncertainties, of each of the input quantities $u(x_i)$. Depending on how the standard uncertainty is estimated, the set of input quantities may be divided into two categories [1,2]:

Evaluation Type A

Standard uncertainty of input quantities can be evaluated in the course of statistical analysis of a series of observations.

Evaluation Type B

The standard uncertainty of input quantities is evaluated by means of tools different from the statistical analysis of a series of observations. In this case, the information can come from the following sources: calibration certificates, handbooks, producers’ specifications and hypotheses on the density function of the input quantity.

Calculating standard uncertainty of the output quantity on the basis of the law of propagation of variances, the expanded uncertainty of measurement U can be obtained when multiplying the standard uncertainty by a coverage factor k:

$$U = k\,u(y). \tag{7}$$

The value of k depends on the probability distribution of the output quantity y and on the level of confidence. The assigned expanded uncertainty corresponds to a coverage probability of approximately 95%. In most of the calibrations the output distribution can approach a normal distribution with $k = 2$.

3. CONTOUR MEASUREMENT METHODS

Complicated surface contours were measured using the surface texture measuring system "Perthometer Concept" produced by company MAHR [[10]]. A simplified schema of the system is shown in Fig. 1 and a photo in Fig. 2.

The Institute of Mechatronics has owned this system for about one year. "Perthometer Concept" is a modular computer-controlled station for measuring and analysing roughness, contour and topography of surfaces. Its software runs under Windows. Operation is therefore quickly learned, easy to understand and compatible with other Windows' applications. PCV 200 contour drive unit with an exchangeable tracing arm was used in our research. The high-precision PCV 200 contour drive unit is a long-distance instrument for the assessment of radii, distances, angles and straightness deviations. The smooth traverse and the computer-assisted error correction guarantee reproducible measurements with utmost vertical and horizontal resolution in a measuring field of 200×50 mm. PCV 200 contour drive unit allows automatic lowering and lifting of the tracing arm with programmable speed and quick positioning. The measuring force can be adjusted from 2 to 120 mN. Rigid design and unique material provide a highly dynamic construction. Drive unit has programmable measuring routines including lowering, lifting and positioning of the tracing arm and selectable measuring speeds.

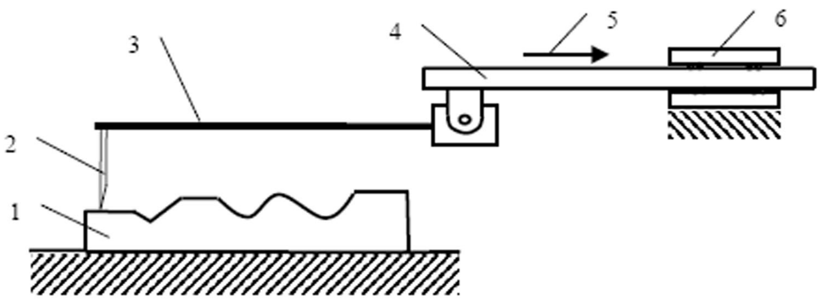

Fig. 1. Schema of "Perthometer Concept": 1 – measuring object; 2 – stylus; 3 – tracing arm; 4 – drive unit; 5 – measuring direction; 6 – calibrated support.

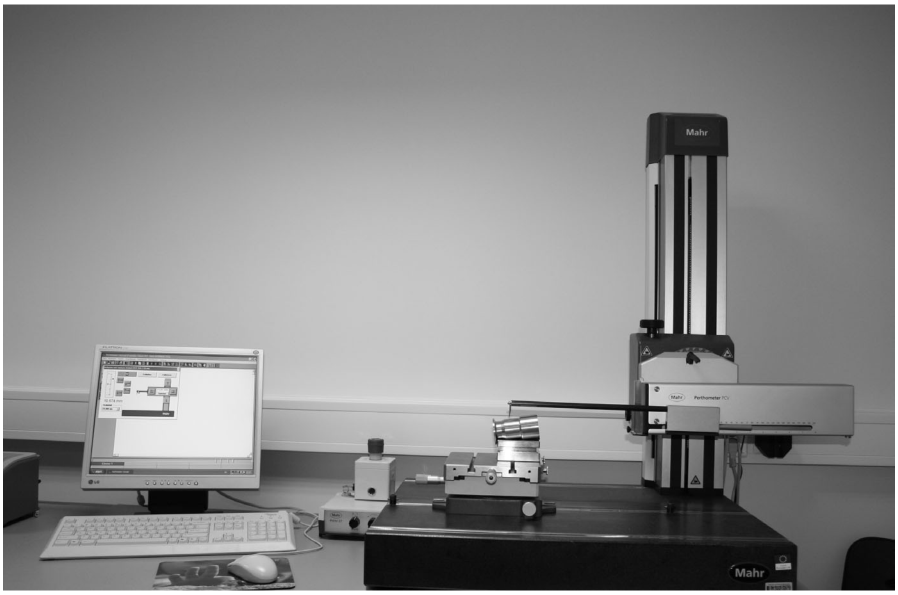

Fig. 2. Surface texture measurement system “Perthometer Concept”.

4. MEASUREMENT MODEL

Proceeding from Eq. (2), the measurement model can be expressed as

$$y = x + \sum_{i=1}^{N-1} \delta x_{N-1}, \tag{8}$$

$$\sum_{i=1}^{N-1} \delta x_{N-1} = \delta x_{\mathrm{MI}} + \delta x_{\mathrm{r}} + \delta x_{\mathrm{F}} + \sum_{j=1}^{J} \delta x_{\mathrm{obj},\, j}, \tag{9}$$

$$\sum_{j=1}^{J} \delta x_{\mathrm{obj},\, j} = \delta x_{\mathrm{cv}} + \delta x_{\mathrm{cc}} + \delta x_{\mathrm{ang}}, \tag{10}$$

where δx_{MI} is the correction from the measuring instrument, δx_{r} is the correction from the stylus radius, δx_{F} is the measurement force correction, δx_{cv} is the surface curvature correction, δx_{cc} is the surface concavity correction and δx_{ang} is the correction from the surface angle.

Now, we can express the measurement model by the following equation:

$$y = x + \delta x_{\mathrm{MI}} + \delta x_{\mathrm{r}} + \delta x_{\mathrm{F}} + \delta x_{\mathrm{cv}} + \delta x_{\mathrm{cc}} + \delta x_{\mathrm{ang}}. \tag{11}$$

5. COMBINED UNCERTAINTY OF A MEASUREMENT RESULT

The standard uncertainty to be ascribed to the estimate y of an output quantity Y, which is evaluated from the estimates of a number of input quantities, is named [1,2] *combined standard uncertainty*. By introducing this concept, it is possible to distinguish the uncertainty of the output quantity from the uncertainty of other quantities that occur in the measurement model. However, the uncertainty of an input quantity is, in its turn, often obtained from a relevant measurement model, which means that during the evaluation process it was itself determined with an uncertainty. Similarly, we can use the output from the measurement model as an input for a measurement task. The concept of combined standard uncertainty is therefore only of limited use. The symbol $u(y)$ is used for the standard uncertainty to be ascribed to the estimate y, regardless of the way in which the uncertainty has been evaluated. The combined standard uncertainty is the positive square root of the combined variance, which is the weighted sum of the experimental variances and covariances of all input quantities considered in the measurement model. The experimental variances and covariances are obtained from the experimental standard deviations $u(x_i)$, associated with the estimates x_i of the input quantities X_i. In our case the combined standard uncertainty is determined as

$$u(y) = [u^2(x) + u^2(\delta x_{\mathrm{MI}}) + u^2(\delta x_{\mathrm{r}}) + u^2(\delta x_{\mathrm{F}}) + u^2(\delta x_{\mathrm{cv}}) + u^2(\delta x_{\mathrm{cc}}) + u^2(\delta x_{\mathrm{ang}})]^{1/2}. \qquad (12)$$

6. RESULTS

Standard uncertainties of input quantities from different sources were determined. The following results were obtained experimentally and their standard uncertainties were calculated applying type B method of uncertainty evaluation.

Indication (contour) x

Indication in this case is a contour we can see on the screen of the computer (Fig. 3). Standard uncertainty of the indication can be determined according to the printer resolution. The current printer resolution is $\Delta x = \pm 1\ \mu\mathrm{m}$:

$$u(x) = \frac{\Delta x}{\sqrt{3}} = 0.6\ \mu\mathrm{m}.$$

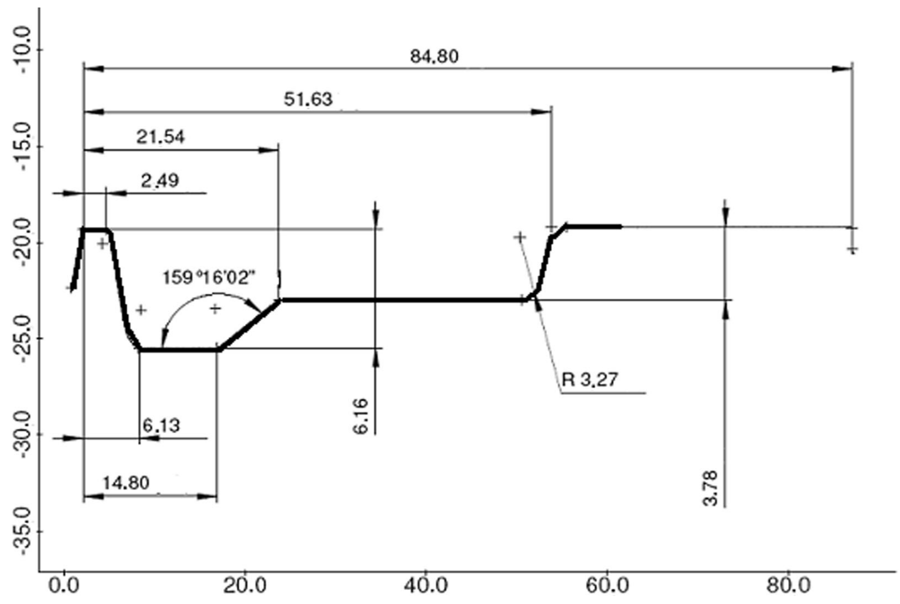

Fig. 3. The contour measured.

Measuring instrument correction δx_{MI}

This correction was not found in the calibration certificate, but it was mentioned, that the indication can change within the limits of $\Delta_{\mathrm{MI}} = \pm 0.5\ \mu\mathrm{m}$:

$$u(\delta x_{\mathrm{MI}}) = \frac{\Delta_{\mathrm{MI}}}{\sqrt{3}} \cong 0.3\ \mu\mathrm{m}.$$

Stylus radius correction δx_{r}

Research results indicated that stylus radius correction does not remarkably affect the contour measurements. Thus we have assumed $\delta x_{\mathrm{r}} \cong 0$ and $u(\delta x_{\mathrm{r}}) \cong 0$.

Measuring force correction δx_{F}

Measuring force correction and its standard uncertainty can be calculated as follows. From the Hertz formula the elastic deformation can be calculated. The worst situation, sphere–sphere, was assumed. The correction value is to be considered equal to zero and its standard uncertainty can be calculated according to the following equation:

$$u(\delta x_{\mathrm{F}}) = \frac{\Delta_{\mathrm{F}}}{\sqrt{3}} \cong 0.3\ \mu\mathrm{m},$$

where $\Delta_{\mathrm{F}} = \pm 0.6\ \mu\mathrm{m}$.

Surface complexity corrections δx_{cv}, δx_{cc}

The corrections due to the surface curvature and concavity are assumed to be equal to zero: $\delta x_{\mathrm{cv}} \cong 0$ and $\delta x_{\mathrm{cc}} \cong 0$.

The standard uncertainty of these corrections can be calculated from the following equations:

$$u(\delta x_{\mathrm{cv}}) = \frac{\Delta_{\mathrm{cv}}}{\sqrt{3}} \cong 0.9\ \mu\mathrm{m},$$

$$u(\delta x_{\mathrm{cc}}) = \frac{\Delta_{\mathrm{cc}}}{\sqrt{3}} \cong 0.9\ \mu\mathrm{m},$$

where Δ_{cv} and Δ_{cc} have been found experimentally: $\Delta_{\mathrm{cc}} = \Delta_{\mathrm{cv}} = \pm 1.5\ \mu\mathrm{m}$.

Correction of the surface angle δx_{ang}

Correction of the surface angle $\delta x_{\mathrm{ang}} = 0$ and its standard uncertainty can be calculated from the equation:

$$u(\delta x_{\mathrm{ang}}) = \frac{\Delta_{\mathrm{ang}}}{\sqrt{3}} \approx 1.7\ \mu\mathrm{m},$$

where Δ_{ang} was experimentally determined during the research applying the angle standards: $\Delta_{\mathrm{ang}} = \pm 2.9\ \mu\mathrm{m}$.

The above-mentioned quantities and their estimations are presented in Table 1.

From Eq. (12) we have

$$u(y) = \sqrt{\sum_{i=1}^{N} u^2(x_i)} = \sqrt{5.05} \cong 2.3\ \mu\mathrm{m}.$$

Hence, the expanded uncertainty can be given, according to Eq. (7), as

$$U = k\,u(y) = 2 \cdot 2.3 = 4.6\ \mu\mathrm{m} \cong 5\ \mu\mathrm{m}.$$

Table 1. Estimates of the input quantities and their uncertainties

Quantity X_i	Estimate x_i	Standard uncertainty $u(x_i)$, µm	Dispersion $u^2(x_i)$
x	contour	0.6	0.36
δx_{MI}	0	0.3	0.09
δx_{r}	0	0	0
δx_{F}	0	0.3	0.09
δx_{cv}	0	0.9	0.81
δx_{cc}	0	0.9	0.81
δx_{ang}	0	1.7	2.89
		$\sum$	5.05

7. CONCLUSIONS

A measurement model and method for calculating the expanded uncertainty of the surface measurement has been elaborated. It is possible to give an estimation to the surface elements obtained in the printout after measuring the contour of a complicated surface. It has been shown how to estimate the variation range and to analyse the limits within which the numerical values of the surface contour can change. Finally, the quality of the measured values can be evaluated and the measurement uncertainty of the latter can be estimated.

ACKNOWLEDGEMENTS

This work was supported by Estonian Ministry of Education targeted financing grant No. 14250503 and Estonian Science Foundation grant No. 6172.

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Mõõtemääramatuse uurimine pinna kontuuri mõõtmisel

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On uuritud mõõtekompleksi "Perthometer Concepti" (MAHR GmbH) abil saadud pinna kontuuri iseloomustavate suuruste usaldatavust. On koostatud mõõtmiste mudel ja eksperimentaalselt määratud mudeli sisendsuuruste väärtused ning nende jaotus, mis võimaldab mõõtetulemuste usaldatavust iseloomustada nende laiendmääramatusega.