Proc. Estonian Acad. Sci. Phys. Math., 2006, **55**, 4, 246–249

Transverse shifts and conservation of angular momentum at reflection and refraction of an electromagnetic wave packet

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Received 25 September 2006

Abstract. A concept of the separate conservation laws of angular momenta and recent calculations of the transverse shifts of a light packet at reflection and refraction are discussed. It is shown that the results of these calculations are the particular cases of the results obtained in previous works of the author. The most general condition under which the shifts satisfy the separate conservation laws is pointed out.

Key words: light packet, reflection, refraction, transverse shift, conservation law.

1. INTRODUCTION

The phenomenon of the transverse shifts (TSs) accompanying the processes of reflection and refraction of the light packets at a plane interface of two isotropic transparent media is now well known. The first calculations of the TS of the totally reflected light beam were based on the Kard method (energy-flux method). The results are presented in [1]. In [2,3], the general calculations of the TSs of the secondary beams are performed. It is shown that the TS phenomenon takes place in the total-reflection as well as in partial-reflection regimes.

Recently, interest in the TS phenomenon was renewed. In papers [4,5], the relation between the TS phenomenon and conservation of the normal to the interface component of the angular momentum (NCAM) of the electromagnetic field is discussed. The conservation law for the NCAM of the total electromagnetic field can be proved rigorously [6,7]. Onoda et al. [4] assumed, in addition, that such conservation laws take also place in the processes of reflection and refraction separately. On the basis of this assumption, they obtained the expressions for the TS of the partially reflected and refracted light beams ([4], Eq. (4)). On the other hand, the authors of [5] directly calculated these shifts in concrete processes, reflection and refraction of the elliptically polarized Gaussian light beam. Their results ([5], Eqs (5) and (6)) generally do not coincide with the results obtained in [4], and they conclude that the TSs of the partially reflected and refracted light beams do not satisfy the separate conservation laws (Eq. (2) in [5]), except for the case when the incident beam is linearly or circularly polarized. Some arguments for and against the concept of the separate conservation laws are presented in [4] and [5].

2. ANALYSIS OF THE SPIN-DEPENDENT TS

In the present communication the results obtained in [4] and [5] are compared with the result obtained in [2,3]. The last result is given by Eq. (15) in [4], which maintains that the TS of every secondary beam consists of two terms; their physical sense is explained in the third and fourth paragraphs of the section "Discussion of results". The first term ($h^{(j1)}$), which is defined by Eqs (16) and (11) (note the obvious misprint in the latter equation: in its left-hand side the letter $\mathcal{G}$ must be replaced by the letter $\mathcal{J}$), is connected with the transformation of the intrinsic spin angular momentum at reflection and refraction; it evidently coincides with Eq. (4) in [4]. Hence, it is the term $h^{(j2)}$, which destroys the separate conservation law of NCAM. On the basis of Eqs (17), (18), and (20) in [3], one can establish the general condition under which the TSs of the partially reflected and refracted light beams satisfy the separate conservation laws: the transverse distance between the centres of gravity of the s- and p-polarized electromagnetic fields inside the incident beam (the quantity $\Delta^{(i)}_{,a}$ in the right-hand side of Eq. (17)) must be equal to zero; the definition of the s- and p-polarized fields is given by Eq. (7) in [3]. (However, it should be underlined that incident beams carrying intrinsic orbital angular momenta are not considered in the present communication, like in [2−5]. Such beams, the TSs of the partially reflected and refracted light beams have been calculated in [8], see also [9].)

It can be easily verified that the result obtained in [5] is a particular case of the results obtained in [3]. Indeed, once the incident beam is defined by Eq. (3) in [5], the transverse distance between the centres of gravity of its fields polarized along the vectors $\mathbf{e_y}$ and $\mathbf{e_X}$ (the vectors $\mathbf{a}$ and $\mathbf{b^{(j)}}$ in [3]) equals to zero. In [3], such fields were termed as a- and b-polarized, respectively, and defined by Eqs (21a) and (21b); in a general case, the transverse distance between their centres of gravity equals the quantity $\gamma^{(i)}_{,a}$ (see Eqs (25) and (26) in [3]). Equation (27) in [3] establishes the relation between the quantities $\Delta^{(i)}_{,a}$ and $\gamma^{(i)}_{,a}$. If $\gamma^{(i)}_{,a} = 0$, the quantity $\Delta^{(i)}_{,a}$ is defined by the second term in the right-hand side of Eq. (27); it can be easily verified that in notations of [5] this quantity looks as follows:

$\Delta^{(i)}_{,a} = \Im m \cot\theta(1 - |m|^2)|m|^{-2}k^{-1}$. In the considered case, the condition, under which the TSs of the partially reflected and refracted light beams satisfy the separate conservation laws, is as follows: $\Im m = 0$ (the linear polarization of the incident beam), or $|m|^2 = 1$ (the circular polarization is the particular case of this condition). Using Eq. (12) in [3], one can write down the difference $Q^{(j)} - Q^{(i)}$ in notations of [5]. Gathering the above-mentioned results in the right-hand side of Eq. (17), one can be convinced that, in the considered case, the term $h^{(j2)}$ equals the difference between the expressions (5) in [5] and (4) in [4].

3. CONCLUSIONS

The results of calculations of the TSs of the secondary beams obtained in [4,5] are the particular cases of the results obtained in [2,3]. The general condition under which the TSs of the secondary beams satisfy the separate conservation law of NCAM is found in [4] as follows:

$$\Delta^{(i)}_{,a} = 0.$$

ACKNOWLEDGEMENT

This work was supported by the Estonian Science Foundation (grant No. 6534).

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Ristnihked ja nurgamomendi jäävus elektromagnetilise lainepaketi peegeldumisel ja murdumisel

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On käsitletud separaatsete impulssmomentide jäävusseaduste kontseptsiooni ja hiljutisi valguspaketi ristnihke arvutusi peegeldumise ning murdumise korral. On näidatud, et nimetatud arvutuste tulemused kujutavad endast autori varasemates töödes saadud tulemuste erijuhtu. On formuleeritud kõige üldisem tingimus, mille korral eelmainitud nihked rahuldavad separaatseid jäävusseadusi.