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Eddy-driven flows over varying bottom topography in natural water bodies

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Abstract. A mechanism of the formation of eddy-driven flows in natural water bodies with varying bottom topography is discussed. The discussion is based on the theory of rotationally anisotropic turbulence. It is argued that a flow develops under the coupled effect of a preferred rotation orientation of turbulent eddies, and bottom topography. The flow formation is illustrated on a simple model. According to the model, flows are formed in regions with the depth smaller than the critical depth predicted by the model.

Key words: turbulence, negative viscosity, currents.

1. INTRODUCTION

Statistical treatment of eddy-driven flows over varying bottom topography has attracted great attention. The methods developed within the approach [1–11] have proved to be productive in the discussion of many aspects of the flows. Nevertheless, the statistical methods are preferred in analysis not only because of their efficiency. It would be more natural to treat many aspects of the flows immediately in terms of average fields, though, the conventional turbulence mechanics (CTM) does not include a tool appropriate for the approach. One substantial shortcoming of the CTM is connected with the "negative viscosity" problem in parameterization of effects associated with the eddy-to-mean energy transfer. Although the notion "negative viscosity", first introduced in [12], has been rather widely used in many investigations [13–15], it is nevertheless physically controversial. The very sense of viscosity as a physical phenomenon does not accept negative viscosity coefficient values, therefore the notion "negative viscosity" should be ascribed a sense of an artificial, physically nonjustified construction to make the theoretical predictions compatible with observational evidence.

Unlike the CTM, the generalized mechanics of turbulence elaborated in [16–18] (henceforth referred to as the theory of rotationally anisotropic turbulence, or the RAT theory) explains the effect without any requirement of the negative viscosity coefficient. The present paper illustrates this assertion on an example of the formation of eddy-driven flows over varying topography.

The formation of eddy-driven flows over varying topography is commonly tied with the potential vorticity conservation [19]. In the suggested model the potential vorticity conservation is derived as a consequence of the moment of momentum or angular momentum (in the sense defined in the RAT theory) conservation for a water column between the surface and the bottom. This treatment assigns the turbulence a permanent role in the flow formation and reduces the problem of the flow energy source to the problem of the energy source of the orientated (large-scale) turbulence constituent. The model explains the orientated nature of the large-scale turbulence constituent as induced by the Earth's rotation. In the model the turbulence feeds the average flow in the regions where the depth is smaller than the critical depth predicted by the model. In the regions of the water body where the actual depth is larger than the critical depth, the actual depth does not influence the medium motion. The motion in this region has a character of two-dimensional turbulence with a preferred rotation orientation of turbulent eddies and zero average flow velocity. The situation agrees with [20], indicating a significant eddy mixing in the ocean interior outside the regions affected by topography.

The paper starts with some basic statements of the RAT theory (Section 2), crucial for understanding the model discussed afterwards (Section 3). The model complements some previous applications of the RAT theory concerning the discussion of oceanographic problems [21–24].

2. THE RAT THEORY

The RAT theory [18] starts from the definition of the quantity

$$\boldsymbol{M} = \left\langle \boldsymbol{v}' \times \boldsymbol{R} \right\rangle, \tag{1}$$

where $\boldsymbol{v}'$ denotes a fluctuating constituent of the flow field velocity $\boldsymbol{v}$, $\boldsymbol{R}$ is the curvature radius of $\boldsymbol{v}'$ streamline, and angular brackets denote statistical averaging. The quantity $\boldsymbol{M}$ differs from zero if turbulent eddies have an average preferred rotation orientation. (Hereafter we call the property of a turbulent flow expressed as $\boldsymbol{M} \neq 0$ the rotational anisotropy.) A preferred rotation orientation of turbulent eddies is expectedly inherent to large-scale eddies, while the small-scale eddies (characterized by a relatively small R), which do not contribute to $\boldsymbol{M}$, constitute rotationally isotropic turbulence. Due to its physical sense as the density of the average moment of momentum (or angular momentum) per unit mass, $\boldsymbol{M}$ is called the internal moment of momentum of the turbulent flow field. Let us stress that the nontriviality of the defined $\boldsymbol{M}$-field presumes the inclusion of $\boldsymbol{R}$ into the arguments of the probability distribution specifying the averaging operation applied.

Consider now some conclusions following from definition (1).

(a) Definition (1) is coupled with the definition of the quantity

$$\boldsymbol{\Omega} = \left\langle \frac{\boldsymbol{v}' \times \boldsymbol{R}}{R^2} \right\rangle, \tag{2}$$

which has the sense of the average angular velocity of rotation of turbulent eddies (henceforth – the angular velocity of internal rotation). As a characteristic of the fluctuating constituent of the turbulent motion field the quantity $\boldsymbol{\Omega}$ differs from the vorticity, $\boldsymbol{\omega} = \frac{1}{2}\nabla \times \boldsymbol{u}$, defined as a characteristic of the average velocity field $\boldsymbol{u} = \langle \boldsymbol{v} \rangle$. The moment $\boldsymbol{M}$ and angular velocity $\boldsymbol{\Omega}$ define the effective moment of inertia J determined by $\boldsymbol{M} = J\boldsymbol{\Omega}$. The square root of J determines the finite spatial scale of the rotationally anisotropic (large-scale) turbulence constituent.

(b) Using the identity $v'^2 = (\boldsymbol{v}' \times \boldsymbol{R}) \cdot (\boldsymbol{v}' \times \boldsymbol{R}) / R^2$ and definitions (1) and (2), the total turbulence energy $K = \frac{1}{2}\langle v'^2 \rangle$ can be presented as

$$K = K^{\Omega} + K^0, \tag{3}$$

where $K^{\Omega} = \frac{1}{2}\boldsymbol{M} \cdot \boldsymbol{\Omega}$ and $K^0 = \frac{1}{2}\langle \boldsymbol{M}' \cdot \boldsymbol{\Omega}' \rangle$ ($\boldsymbol{M}' = \boldsymbol{v}' \times \boldsymbol{R} - \boldsymbol{M}$ and $\boldsymbol{\Omega}' = \boldsymbol{v}' \times \boldsymbol{R} / R^2 - \boldsymbol{\Omega}$) determine the densities (per unit mass) of energies of rotationally anisotropic (large-scale) and rotationally isotropic (small-scale) turbulence constituents.

(c) The description of the motion of the rotationally anisotropic turbulent flow (realized by the RAT theory) is based on the conservation laws of momentum (the Reynolds equation), the moment of momentum $\boldsymbol{M}$, and the turbulence energy K^0. The inclusion of the equation for energy K^0 into the motion description set-up consolidates the RAT theory with the CTM specified as describing the turbulence constituent with energy K^0, and the inclusion of the equation for the moment $\boldsymbol{M}$ relates the RAT theory to the approach of the 1970s [25–27], based on the hydrodynamics of micropolar continua [28–31].

(d) The evident interaction between the average flow and the large-scale turbulence constituent characterized by $\boldsymbol{M}$ turns the turbulent (Reynolds) stress tensor asymmetric with the antisymmetric constituent describing the interaction. The asymmetry becomes possible due to the inclusion of $\boldsymbol{R}$ into the arguments of the probability distribution. In general, the probability distribution appears noninvariant under the commutation of the components of the fluctuating constituent of the velocity field. The specifics of the RAT theory vanish if the averaging is assumed in the sense of the CTM.

3. THE MODEL OF EDDY-DRIVEN FLOW OVER VARYING TOPOGRAPHY IN NATURAL WATER BODIES

3.1. The model set-up

According to [32,33], the dominant interaction between the average flow and the turbulence realizes between the average flow and the large-scale turbulence constituent, characterized within the RAT theory by $\boldsymbol{M}$, therefore the energy equation for K^0 is disregarded in the first approximation of the motion description set-up. However, this equation remains essential for considering the energy processes. Within this approximation the motion description realizes in the frame of the conservation laws for the momentum (the Reynolds equation) and for the moment of momentum $\boldsymbol{M}$ (the equation of the large-scale turbulence) only. These laws are expressed as [18]

$$\rho \frac{\mathrm{d}\boldsymbol{u}}{\mathrm{d}t} = \{\sigma_{ij,j}\} + \rho \boldsymbol{f} \tag{4}$$

and

$$\rho \frac{\mathrm{d}\boldsymbol{M}}{\mathrm{d}t} = \{m_{ij,j}\} - \boldsymbol{\sigma} + \rho \boldsymbol{m}. \tag{5}$$

In (4) and (5) (in addition to $\boldsymbol{M}$ and $\boldsymbol{u}$ explained above) the following notations are used: ρ denotes the medium density; $\mathrm{d}/\mathrm{d}t = \partial/\partial t + \boldsymbol{u}\cdot\nabla$; σ_{ij} and m_{ij} (specified below for the model considered) denote the components of the stress tensor and the moment stress tensor; the notations in curly brackets denote component presentation of the corresponding quantities, the Latin indices obtain the values 1,2,3, the index next to comma denotes differentiation by the respective space co-ordinate, and Einstein summation is assumed; $\boldsymbol{f}$ denotes the (average) body force density per unit mass (henceforth – the body force);

$$\boldsymbol{\sigma} = 4\gamma(\boldsymbol{\Omega} - \boldsymbol{\omega}) \tag{6}$$

is the dual vector of the antisymmetric constituent of the stress tensor (the components of $\boldsymbol{\sigma}$ are defined as $\sigma_k = e_{ijk}\sigma_{ij}$, where e_{ijk} denotes the components of the Levi–Civita tensor; $\gamma > 0$ is the coefficient of the rotational viscosity characterizing the shear in relative rotation, i.e. if $\boldsymbol{\Omega} \neq \boldsymbol{\omega}$), and

$$\boldsymbol{m} = \boldsymbol{m}_\mathrm{f} + \boldsymbol{m}_1 + J(\nabla\boldsymbol{u})\cdot\boldsymbol{\Omega}. \tag{7}$$

In (7)

$$\boldsymbol{m}_\mathrm{f} = \langle \boldsymbol{f}' \times \boldsymbol{R} \rangle \tag{8}$$

is the moment caused by the body force fluctuations $(\boldsymbol{f}')$ and

$$\boldsymbol{m}_1 = -4\frac{\kappa}{\rho}\boldsymbol{\Omega} \tag{9}$$

is the moment caused by the decay of the average effect of eddies rotation orientation in the cascading process ($\kappa > 0$ is the coefficient of cascade scattering; further, κ as well as γ and J defined above are considered constant).

The quantities $\boldsymbol{f}$ and $\boldsymbol{m}_\mathrm{f}$ in (4) and (7) are specified as dependent on the external fields acting on the medium. In particular, for the motion of the medium with constant density in the gravity field we have $\boldsymbol{f} = \boldsymbol{g}$, where $\boldsymbol{g}$ is the gravitational acceleration, and $\boldsymbol{m}_\mathrm{f} = 0$. In geophysical situation, considering the motion in a frame rotating with the Earth, Eq. (4) is modified by replacing $\boldsymbol{f}$ with

$$\boldsymbol{f} = \boldsymbol{g} + 2\boldsymbol{u}\times\boldsymbol{\omega}^0, \tag{10}$$

where $\boldsymbol{\omega}^0$ is the angular velocity of the Earth's rotation, and the second term on the right side of (10) is the density per unit mass of the Coriolis force. Equation (5) is also modified by replacing $\mathrm{d}\boldsymbol{M}/\mathrm{d}t$, $\boldsymbol{\Omega}$, $\boldsymbol{\omega}$, and $J(\nabla\boldsymbol{u})\cdot\boldsymbol{\Omega}$ with $\mathrm{d}\boldsymbol{M}/\mathrm{d}t + \boldsymbol{\omega}^0\times\boldsymbol{M}$, $\boldsymbol{\Omega}+\boldsymbol{\omega}^0$, $\boldsymbol{\omega}+\boldsymbol{\omega}^0$, and $J(\nabla\boldsymbol{u})\cdot(\boldsymbol{\Omega}+\boldsymbol{\omega}^0) + J\boldsymbol{\Omega}\times\boldsymbol{\omega}^0$, respectively. The replacements performed are equivalent to the replacement of expression (7) for $\boldsymbol{m}$ with

$$\boldsymbol{m} = -4\frac{\kappa}{\rho}(\boldsymbol{\Omega}+\boldsymbol{\omega}^0) + J(\nabla\boldsymbol{u})\cdot(\boldsymbol{\Omega}+\boldsymbol{\omega}^0) + 2J\boldsymbol{\Omega}\times\boldsymbol{\omega}^0. \tag{11}$$

Let us apply now Eqs (4) and (5), together with $\boldsymbol{f}$ and $\boldsymbol{m}$ determined in (10) and (11), to a natural water body in the northern hemisphere with depth $H = H(x, y)$ (the right-hand co-ordinate system (x, y, z) with the vertical axis z directed downwards and with $z = 0$ on the free surface is used). The following assumptions are adopted:

(i) $\boldsymbol{\omega}^0$ is identified with the vertical projection of the angular velocity of the Earth's rotation;

(ii) the flow field velocity $\boldsymbol{u}$ can be presented as $\boldsymbol{u} = (u_x(x, y), u_y(x, y), 0)$ everywhere except in the continuity equation $(\nabla\cdot\boldsymbol{u} = 0)$ expressed as [34]

$$\frac{\partial u_z}{\partial z} = \frac{1}{H}\frac{\mathrm{d}H}{\mathrm{d}t} = \frac{1}{H}\boldsymbol{u}\cdot\nabla_h H \tag{12}$$

$(\nabla_h = (\partial/\partial x, \partial/\partial y, 0)$ and $\mathrm{d}/\mathrm{d}t \equiv \boldsymbol{u}\cdot\nabla_{\boldsymbol{h}})$;

(iii) $\boldsymbol{\Omega} = (0, 0, \Omega(x, y))$;

(iv) the effects caused by the turbulent shear stresses (described by the symmetric constituent of the stress tensor and realizing the average flow interaction with the small-scale turbulence) are negligibly small with respect to the effects caused by the antisymmetric constituent of stresses (describing the average flow interaction with large-scale turbulence), i.e.

$$\{\sigma_{ij}\} = -p\hat{\mathbf{1}} + \mathsf{E}\cdot\boldsymbol{\sigma}, \tag{13}$$

where p, $\hat{\mathbf{1}}$, and E denote thermodynamic pressure, the unit tensor, and the Levi–Civita tensor;

(v) the diffusive effects of the moment of momentum, described by the moment stress tensor m_{ij} in (4), can be ignored, i.e. m_{ij} vanishes.

Within the model specifications made it follows from Eqs (4) and (5) that $\partial p/\partial z = \rho g$,

$$\rho\frac{\mathrm{d}\boldsymbol{u}}{\mathrm{d}t} = -\nabla_h p + 2\gamma\nabla_h \times(\boldsymbol{\Omega}-\boldsymbol{\omega}) + 2\rho\boldsymbol{u}\times\boldsymbol{\omega}^0, \tag{14}$$

and

$$\rho J\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{\Omega} = -4\gamma(\boldsymbol{\Omega}-\boldsymbol{\omega}) - 4\kappa(\boldsymbol{\Omega}+\boldsymbol{\omega}^0) + \rho J\frac{\partial u_z}{\partial z}(\boldsymbol{\Omega}+\boldsymbol{\omega}^0). \tag{15}$$

From (14) and (15), using the continuity equation in form (12), we have

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\boldsymbol{\omega}+\boldsymbol{\omega}^0}{H} = \frac{\gamma}{H}\nabla_h\times\nabla_h\times(\boldsymbol{\Omega}-\boldsymbol{\omega}) \tag{16}$$

and

$$\rho J\frac{\mathrm{d}}{\mathrm{d}t}\frac{\boldsymbol{\Omega}+\boldsymbol{\omega}^0}{H} = -4\gamma\frac{\boldsymbol{\Omega}-\boldsymbol{\omega}}{H} - 4\kappa\frac{\boldsymbol{\Omega}+\boldsymbol{\omega}^0}{H}, \tag{17}$$

where $(\boldsymbol{\omega}+\boldsymbol{\omega}^0)/H$ is the potential vorticity.

3.2. Model analysis

The model analysis is started from Eqs (16) and (17). Consider first the situation when $\boldsymbol{u}\equiv 0.$ Then from (17) it follows that

$$\boldsymbol{\Omega} = -\frac{\kappa}{\gamma+\kappa}\boldsymbol{\omega}^0. \tag{18}$$

Expression (18) states that there are rotating eddies in the flow, although the mean flow is absent. (Depth H drops out of the description due to $\boldsymbol{u}\equiv 0,$ because of which H is not constrained in any way.) This corollary of the model agrees with [20], concluding a significant eddy mixing in the ocean interior outside the regions affected by topography, and with [21], where the $\boldsymbol{\Omega}$-field is considered as being essential for the salinity transport in the ocean. The effect declared by (18) does not come up for $\kappa = 0,$ i.e. if the cascade scattering of the moment of momentum is hampered. When $\gamma = 0,$ i.e. if the shear in the relative rotation is absent, then $\boldsymbol{\Omega} = -\boldsymbol{\omega}^0$ and the medium remains resting in the absolute (nonrotating) frame. (The energetic background of (18) is explained by the energy balance condition (27) below.)

Consider now the situation when $\boldsymbol{u} \neq 0,$ $\boldsymbol{\omega} \neq 0.$ Let us proceed from the presumption that the moment of momentum $J(\boldsymbol{\Omega} + \boldsymbol{\omega}^0)$ is conserved for the water columns extended between the surface and the bottom, expressed mathematically as

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\boldsymbol{\Omega} + \boldsymbol{\omega}^0}{H} = 0. \tag{19}$$

If condition (19) holds, then it follows from Eq. (17) that

$$\boldsymbol{\omega} + \boldsymbol{\omega}^0 = \frac{\gamma + \kappa}{\gamma}(\boldsymbol{\Omega} + \boldsymbol{\omega}^0) \tag{20}$$

and the conservation of the potential vorticity is an immediate result of (19).

Due to $\boldsymbol{u} \neq 0,$ from (19) it follows that

$$\boldsymbol{\Omega} = \boldsymbol{C}H - \boldsymbol{\omega}^0, \tag{21}$$

where $\boldsymbol{C}$ is a constant vector. Using (21), we have from (20)

$$\boldsymbol{\omega} = \frac{\gamma + \kappa}{\gamma}\boldsymbol{C}H - \boldsymbol{\omega}^0. \tag{22}$$

To determine $\boldsymbol{C},$ the balance of energies $K^u = \frac{1}{2}u^2$ and K^{Ω} is considered. Equations for K^u and K^{Ω} follow from (14) and (15) after their scalar multiplication by $\boldsymbol{u}$ and $\boldsymbol{\Omega},$ respectively. Restricting ourselves to the situation when $\boldsymbol{\omega}^0 = \mathrm{const},$ i.e. neglecting the latitudinal variance of $\boldsymbol{\omega}^0,$ we have

$$\rho\frac{\mathrm{d}K^u}{\mathrm{d}t} = \nabla_h \cdot \boldsymbol{h}^u - Q, \tag{23}$$

$$\rho\frac{\mathrm{d}K^{\Omega}}{\mathrm{d}t} = Q - A - \Psi - B. \tag{24}$$

In (23) and (24), $\boldsymbol{h}^u = -p\boldsymbol{u} + 2\gamma(\boldsymbol{\Omega} - \boldsymbol{\omega}) \times \boldsymbol{u}$ denote the flux vector for energy K^u;

$$Q = -4\gamma(\boldsymbol{\Omega} - \boldsymbol{\omega}) \cdot \boldsymbol{\omega} \tag{25}$$

describes the interaction between energies K^u and K^{Ω};

$$\Psi = 4\kappa\Omega^2 + 4\gamma\,|\,\boldsymbol{\Omega} - \boldsymbol{\omega}\,|^2 > 0 \tag{26}$$

describes the scatter of energy K^{Ω} into energy K^0; $A = 4\kappa\boldsymbol{\Omega}\cdot\boldsymbol{\omega}^0$ and $B = -\rho J(\boldsymbol{\Omega}+\boldsymbol{\omega}^0)\cdot\boldsymbol{\Omega}H^{-1}\,\mathrm{d}H/\mathrm{d}t$ describe the interaction of energy K^{Ω} with K^0, their signs depend on the specific situation. (The given interpretation of Ψ, A, and B follows from the comparison of Eq. (24) with the equation for K^0, following as the difference of the energy equations for total turbulence K and for K^{Ω}.) So, as condition (19) reads in energetic terms as $\rho\mathrm{d}K^{\Omega}/\mathrm{d}t = -B$, from (24) we have

$$Q = A + \Psi. \tag{27}$$

Consider now the quantity Ψ. Replacing $\boldsymbol{\Omega}$ and $\boldsymbol{\omega}$ in (26) according to (21) and (22) and determining the critical depth H_{cr} as the depth where $\mathrm{d}\Psi/\mathrm{d}H = 0$, i.e. where the scatter of energy K^{Ω} into energy K^0 obtains the minimum value, we have for C,

$$\boldsymbol{C} = \frac{\gamma}{\gamma+\kappa}\frac{1}{H_{\mathrm{cr}}}\boldsymbol{\omega}^0. \tag{28}$$

The achieved minimum value of Ψ is determined as $4\gamma\kappa(\gamma+\kappa)^{-1}\omega^{0^2}$ and it differs from zero if γ and κ differ from zero. Making use of (28), we have from (21), (22), and (25)

$$\boldsymbol{\Omega} = -\left(1 - \frac{\gamma}{\gamma+\kappa}\frac{H}{H_{\mathrm{cr}}}\right)\boldsymbol{\omega}^0, \tag{29}$$

$$\boldsymbol{\omega} = -\left(1 - \frac{H}{H_{\mathrm{cr}}}\right)\boldsymbol{\omega}^0, \tag{30}$$

and

$$Q = -4\frac{\gamma\kappa}{\gamma+\kappa}\frac{H}{H_{\mathrm{cr}}}\cdot\left(1 - \frac{H}{H_{\mathrm{cr}}}\right)\omega^{0^2}. \tag{31}$$

From the comparison of (29) with (18) it follows that (18) is equivalent to the replacement of the actual depth H with the effective depth equalized with H_{cr}. In the area where the equality (18) holds (coinciding with the area outside the regions affected by topography), the actual depth does not influence the flow, Ψ has a minimum, and $Q = 0$. In this area the actual depth H becomes essential only if there is influx of energy K^u through the boundaries (caused, for example, by the wind stress at the upper boundary of the water body). Then Q becomes positive to compensate the energy influx and, according to (31), $H_{\mathrm{cr}} < H$.

Consider now the area with $H < H_{\mathrm{cr}}$, where, according to (31), $Q < 0$, i.e. the turbulence energy feeds the average flow. This is the situation, for example, in shallow coastal regions. On the basis of (30) it can be concluded that in these regions the flow is directed anticyclonically in a closed water body and cyclonically around islands. Let us stress the essential role of the conditions $\kappa \neq 0$ and $\gamma \neq 0$ for the suggested model, i.e. the effect is predicted as an effect specific to the RAT theory. Indeed, in the case of the CTM we have $\kappa = 0$ or $\gamma = 0$ and therefore $Q = \Psi = 0$. The critical depth H_{cr} as well as $\boldsymbol{\omega}$ and $\boldsymbol{\Omega}$ would remain undetermined in this case. Finally, H_{cr} can be estimated in real flows by the behaviour of the covariance between the velocity fluctuation components: for $H_{\mathrm{cr}} < H$ the covariance is constant, while for $H < H_{\mathrm{cr}}$ the covariance changes together with H.

4. CONCLUSIONS

The suggested model predicts the following flow scheme. Suppose the energy fluxes to the average flow through the water body boundaries are excluded; then the average flow velocity is zero in the interior of the water body outside the regions affected by bottom topography. The scheme corresponds to the minimum scatter of the turbulence energy K^{Ω} into energy K^{0}, compensated by the opposite action of the Earth's rotation providing the turbulence with the property of rotational anisotropy. Due to this balance there is a motion in the form of eddy rotation in the water body, although the motion in the form of the average flow is absent. In the area where the actual depth is smaller than the critical depth (like in the shallow coastal area), the bottom topography becomes essential and the medium turbulence appears to feed the average flow energy. The effect can be explained as a specific effect predicted by the RAT theory.

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REFERENCES

1. Salmon, R., Holloway, G. and Henderschott, M. C. The equilibrium statistical mechanics of simple quasi-geostrophic models. *J. Fluid Mech.*, 1976, **75**, 691–703.
2. Bretherton, F. P. and Haidvogel, D. B. Two dimensional turbulence above topography. *J. Fluid Mech.*, 1976, **78**, 129–154.
3. Herring, J. R. On the statistical theory of two dimensional topographic turbulence. *J. Atmos. Sci.*, 1977, **34**, 1731–1750.
4. Holloway, G. A spectral theory of non linear barotropic motion above irregular topography. *J. Phys. Oceanogr.*, 1978, **8**, 414–427.
5. Holloway, G. Systematic forcing of large-scale geophysical flows by eddy–topography interaction. *J. Fluid Mech.*, 1987, **184**, 463–476.
6. Haidvogel, D. B. and Brink, D. H. Mean currents driven by topographic drag over the continental shelf and slope. *J. Phys. Oceanogr.*, 1986, **16**, 2159–2171.
7. Carnavale, G. F. and Frederiksen, J. S. Nonlinear stability and statistical mechanics of flow over topography. *J. Fluid Mech.*, 1987, **175**, 157–181.

8. Holloway, G. and Sou, T. Measuring skill of a topographic stress parameterization in a large-scale ocean model. *J. Phys. Oceanogr.*, 1996, **26**, 1088–1092.
9. Adcock, S. T. and Marshall, D. P. Interactions between geostrophic eddies and the mean circulation over large-scale bottom topography. *J. Phys. Oceanogr.*, 2000, **30**, 3232–3238.
10. Polyakov, I. An eddy parameterization based on maximum entropy production with application to modeling of the Arctic Ocean circulation. *J. Phys. Oceanogr.*, 2001, **31**, 2255–2270.
11. Merryfield, W. J., Cummins, P. F. and Holloway, G. Equilibrium statistical mechanics of barotropic flow over finite topography. *J. Phys. Oceanogr.*, 2001, **31**, 1880–1890.
12. Starr, V. P. *Physics of Negative Viscosity Phenomena*. McGraw-Hill, New York, 1968.
13. Bacri, J.-C. and Perzynski, R. “Negative-Viscosity” effect in a magnetic fluid. *Phys. Rev. Lett.*, 1995, **75**, 2128–2131.
14. Morimoto, H., Maekawa, T. and Matsumoto, Y. Nonequilibrium Brownian dynamics of negative viscosity induced in a magnetic fluid subjected to both ac magnetic and shear flow fields. *Phys. Rev. E*, 2002, **65**, 061508.
15. Smolyakov, A. I., Diamond, P. H. and Malkov, M. Coherent structure phenomena in drift wave–zonal flow turbulence. *Phys. Rev. Lett.*, 2000, **84**, 491–494.
16. Heinloo, J. *Phenomenological Mechanics of Turbulence*. Valgus, Tallinn, 1984 (in Russian).
17. Heinloo, J. *Turbulence Mechanics. Introduction to the General Theory of Turbulence*. Estonian Academy of Sciences, Tallinn, 1999 (in Russian).
18. Heinloo, J. The formulation of turbulence mechanics. *Phys. Rev. E*, 2004, **69**, 056317.
19. Rhines, P. B. and Holland, W. R. A theoretical discussion of eddy-driven mean flows. *Dyn. Atmos. Oceans*, 1979, **3**, 289–325.
20. O’Dwyer, J., Williams, R. G., Lacasce, J. H. and Speer, K. G. Does the potential vorticity distribution constrain the spreading of floats in the North Atlantic? *J. Phys. Oceanogr.*, 2000, **30**, 721–732.
21. Toompuu, A., Heinloo, J. and Soomere, T. Modelling of the Gibraltar Salinity Anomaly. *Oceanology*, 1989, **29**, 698–702.
22. Heinloo, J. and Võsumaa, Ü. Rotationally anisotropic turbulence in the sea. *Ann. Geophys.*, 1992, **10**, 708–715.
23. Võsumaa, Ü. and Heinloo, J. Evolution model of the vertical structure of the active layer of the sea. *J. Geophys. Res.*, 1996, **101**, 25635–25646.
24. Heinloo, J. and Toompuu, A. Antarctic Circumpolar Current as a density-driven flow. *Proc. Estonian Acad. Sci. Phys. Math.*, 2004, **53**, 252–265.
25. Eringen, A. C. Micromorphic description of turbulent channel flow. *J. Math. Anal. Appl.*, 1972, **39**, 253–266.
26. Eringen, A. C. and Chang, T. S. Micropolar description of hydrodynamic turbulence. *Recent Adv. Engng. Sci.*, 1968, **5**, 1–8.
27. Peddieson, J. An application of the micropolar fluid model to the calculation of turbulent shear flow. *Int. J. Eng. Sci.*, 1972, **10**, 23–32.
28. Eringen, A. C. Theory of micropolar fluids. *Math. Mech.*, 1966, **16**, 1–18.
29. Dahler, J. S. Transport phenomena in a fluid composed of diatomic molecules. *J. Chem. Phys.*, 1959, **90**, 1447–1475.
30. Dahler, J. S. and Scriven, L. F. Angular momentum of continua. *Nature*, 1961, **192**, 36–37.
31. Eringen, A. C. Mechanics of micromorphic continua. In *Mechanics of Generalized Continua* (Kröner, E., ed.). Springer-Verlag, 1968, 18–33.
32. Richardson, L. F. *Weather Prediction by Numerical Process*. Cambridge University Press, Cambridge, 1922.
33. Kolmogoroff, A. N. The local structure of turbulence in incompressible viscous fluids for very large Reynolds numbers. *C. R. Acad. Sci. URSS*, 1941, **30**, 376–387.
34. Pedlosky, J. *Geophysical Fluid Dynamics*. Springer, New York, 1987.

Keeriste poolt tekitatud voolamised muutuva topograafia kohal looduslikes veekogudes

Jaak Heinloo

On käsitletud keeriste pöörlemise eelisorientatsiooni ja veekogu põhja topograafia koosmõjul tekkivate voolamiste kujunemist, lähtudes pöördeliselt mitte-isotroopsete keskkondade mehaanikast. On näidatud, et veekogu põhja topograafia ja keeriste pöörlemise vastasmõju genereerib pikivoolamisi piirkondades, kus veekogu sügavus on väiksem teatud teooria poolt määratud iseloomulikust sügavusest.