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MATHEMATICAL
MODELLING

Mathematical modelling of the performance of a twisting-ball display

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Abstract. A mathematical model of an electrophoretic information display is considered. A system of differential equations describing the behaviour of an elementary cell of the twisting-ball display is introduced. A theoretical overview of the theories of ball shift and rotation is given. Results of numerical experiments of modelling the balls with different physical parameters are presented. The system of equations was solved using MATLAB solvers implementing Runge–Kutta methods with variable time step using step-wise integration. The display performance function describing the dependence between luminance and rotation in cases of different physical parameters is developed.

Key words: twisting-ball display, mathematical model, dipole movement.

1. INTRODUCTION

A twisting-ball display (Sheridon, 1978; Sheridon and Richley, 1999) is a kind of electrophoretic information display invented at the Xerox Palo Alto Research Center (PARC), called Gyricon. This kind of display consists of a thin layer of transparent silicone plastic in which multiple bichromal balls are randomly dispersed (Fig. 1).

Each ball is an electrical dipole and is placed in the cavity filled with dielectric fluid. The width of the cavity is 10–30% greater than the diameter of the balls. When the polarity of the control voltage is changed the orientation of the balls changes; one or the other of the hemispheres with different charge and colour is exposed to the viewer. The displays based on such physical principles are highly bi-stable, robust, easy to manufacture, and have very low power consumption as they do not emit light because the image is formed using ambient light, similarly to conventional printed paper.

The active elements (balls) of Xerox Gyricon displays are made of different waxes and are charged by a stochastic process that causes each particle to have a somewhat different charge (Crowley et al., 2002).

Because of this the dipole charges of the balls are low and unequal and the required control voltages are very high (Sheridon, 2005). Moreover, the quality of the image is not sufficiently good. The authors of this work developed a method for manufacturing the particles of polyvinylidene difluoride (PVDF) (Liiv, 2013), which is an electret material with an extremely high residual

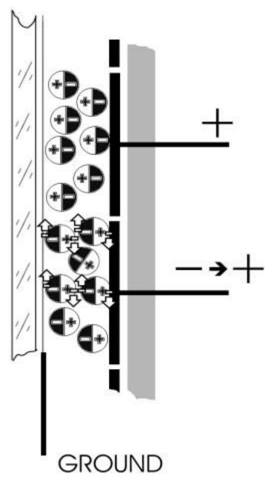

Fig. 1. Schematic cross-section of twisting balls based e-paper display.

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electric field. An electret material is a stable dielectric with a permanently embedded static electric charge, which, owing to the high resistance of the material, will not decay for hundreds of years. The balls are made by first melting or heating a suitable dielectric material and then allowing it to cool in a powerful electrostatic field. The polar molecules of the dielectric align themselves in the direction of the electrostatic field, producing a permanent electrostatic bias. This allows a decrease of the control voltage and an improvement of the image quality.

When the control voltage is constant or zero, the ball is "glued" to the wall of the cavity due to the electrostatic forces. When the polarity of the control voltage changes, the ball begins to move towards the opposite wall of the cavity. Microscopic asymmetries and the "rolling effect" cause a deviation of the axis of the electrical dipole from the direction of the electrical field. Then an electrostatic torque appears and causes the ball to rotate.

This paper describes a simplified mathematical model of a composite display consisting of multiple elements and provides an opportunity to determine the performance of the display (control voltage–luminance) depending on the physical parameters of the balls.

2. THEORY OF THE MOVEMENT OF THE BALLS

When the polarity of the control voltage changes, the ball begins to move towards the opposite wall of the cavity and rotate. A schematic view of the situation is given in Fig. 2.

Notations used:

r – radius of the ball;
s – thickness of the elastomer sheet;
n – coefficient of expansion of the elastomer sheet;
r_c – radius of the cavity;
U_c – control voltage;
ρ – density of PVDF;
q – monopolar charge of the particle;
q_D – dipole charge;
r_D – radius of the dipole;
$\acute{\eta}$ – viscosity of the fluid.

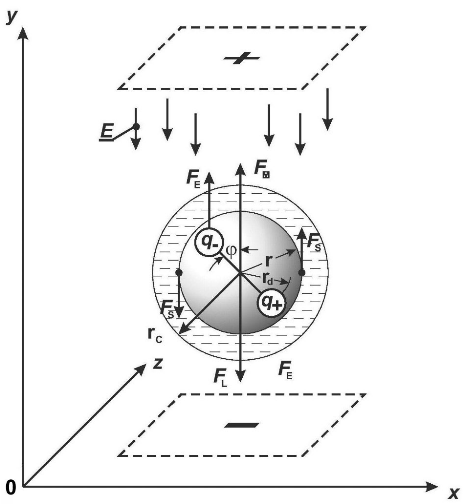

Fig. 2. Schematic view of a bichromatic ball in a cavity.

For the radius of the cavity we have

$$r_c = \alpha \cdot r \tag{1}$$

with some $\alpha > 1$. For the radius of the dipole we have

$$r_D = \frac{r}{2}. \tag{2}$$

The physical bounds for ball shift are $0 \le y(t) \le 2(r_c - r)$. For the simulation, we apply the electric force in the form of a periodic rectangle function $U(t)$ with period T and amplitude $\pm U_c$.

The integral Lambertian luminance of this kind of display is very complicated. The authors are currently developing a model of integral luminance, which considers the stochastic placement of the particles and diffraction, refraction, and reflection of light on different surfaces inside the display. However, here we use a very simplified model as we need only a relative value of the luminance to describe the performance of the display (dependence luminance–position), not the absolute one.

We presume that all the particles are placed in a regular way (see Fig. 3). We presume that the reflectance of the black side of a particle is 0 and of the white side is 100%.

For modelling the reflection, the plane of the display is divided into hexagons with cavities inside these as shown in Fig. 3.

The area of the visible size of the particle is

$$S_1 = \pi \cdot r^2. \tag{3}$$

The area of the corresponding hexagon is

$$S_2 = \frac{3r_c^2}{\sin 60^\circ} = \frac{2}{\sqrt{3}} r_c^2 = \frac{2}{\sqrt{3}} n^2 \cdot r^2. \tag{4}$$

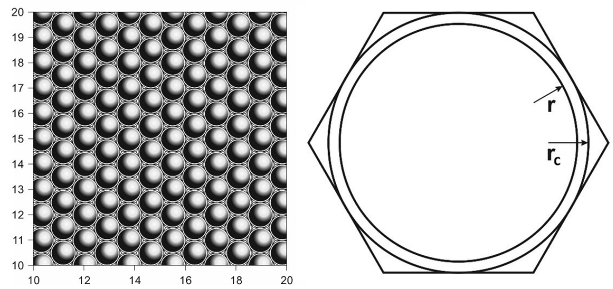

Fig. 3. The plane of the display is divided into hexagons; r – radius of the ball, r_c – radius of the cavity.

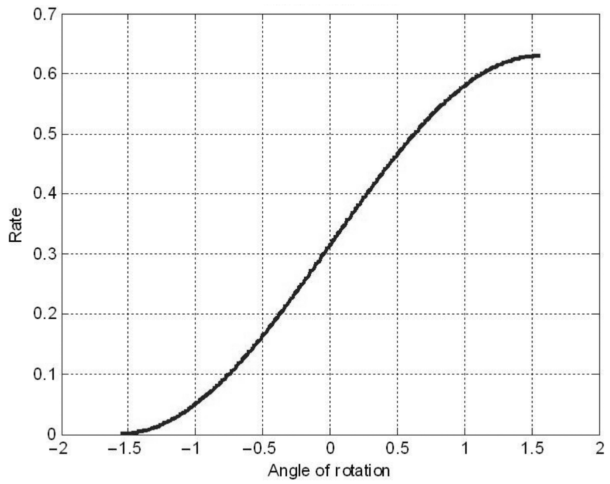

Fig. 4. Rate of the dark visible area of the ball.

The black area of the ball exposed to the viewer is (Seidelmann, 2005)

$$S_3 = \frac{\cos\varphi + 1}{2} S_1. \tag{5}$$

The relative luminance is

$$L = \frac{S_2 - S_3}{S_2} = 1 - \frac{\sqrt{3}}{4} n^2 \cdot (\cos\varphi + 1). \tag{6}$$

The illumination depends on the rotation angle of the multicolour balls inside the cavity. This dependence is depicted in Fig. 4, showing the rate of the dark area as a function of the rotation angle.

3. SOURCE DATA

Most of the source data have a technological background.

The median radius for achieving sufficient image quality (150 dpi) is

$$r = 25\times10^{-6}\ [\text{m}].$$

A larger thickness of the elastomer sheet increases the contrast of the image. At the same time, increasing the thickness means using higher control voltages, which is inadvisable. The optimal thickness achieved experimentally is equal to 3 diameters of the ball:

$$R_{\text{D}} = 150\times10^{-6}\ [\text{m}].$$

The coefficient of expansion of the elastomer sheet n can be between 1.1 and 1.5. In our work we use

$$n = 1.2.$$

The possible control voltage is different for different backplane units. Ideally (using standard solutions)

$$U_{\text{c}} = 15\,[\text{V}].$$

We observe the maximum possible control voltage for commercial backplanes:

$$U_{\text{c}} = 100\,[\text{V}].$$

The density of PVDF is given by (Solvay, n.d.)

$$\rho = 1.75\times10^{3}\ [\text{kg/m}^3].$$

In our experiments, the dipole charge can be controlled by changing the polarization parameters and the monopolar charge can be simply controlled using nonpolar surfactants dissolved in the carrier liquid (Karvar et al., 2011). The values can be measured directly

$$q_{\text{D}} = 1\times10^{-18}\ [\text{C}] \text{ to } 1\times10^{-16}\ [\text{C}],$$
$$q = 1\times10^{-18}\ [\text{C}] \text{ to } 1\times10^{-16}\ [\text{C}].$$

Hexane is used as the carrier liquid. The viscosity of hexane (Sigma-Aldrich, n.d.) is

$$\acute{\eta} = 3\times10^{-4}\ [\text{N}\times\text{s/m}^2].$$

4. MATHEMATICAL MODELLING

4.1. Translation of the ball

The ball inside the cavity accelerates at first and reaches a stable velocity determined by the diameter of the ball and the viscosity of the carrier liquid. Without loss of generality we can assume that the equilibrium state of a ball in the cavity is at the cavity wall. Each ball has a monopolar electrical charge and a bipolar charge. When the control voltage is constant or zero, the ball is “glued” to the wall due to the electrostatic forces. If the control voltage changes, the ball will begin to move towards the opposite wall of the cavity. We neglect the influence of gravity and buoyant force because of their smallness in comparison with the electrostatic force and viscous drag. The shift $y = y(t)$ of the ball is described by the differential equation

$$\frac{d^2y}{dt^2} - \frac{F_{\text{E}} - F_{\text{L}}}{m} = 0. \tag{7}$$

Here, F_{E} is an electrostatic force

$$F_{\text{E}} = E \cdot q \tag{8}$$

and F_{L} is a viscous drag. We presume that the velocity of the ball is relatively low and we can use Stokes’s law to determine the resistance of the fluid:

$$F_{\text{L}} = d \cdot v = 6 \cdot \pi \cdot \eta \cdot r \cdot \frac{dy}{dt}. \tag{9}$$

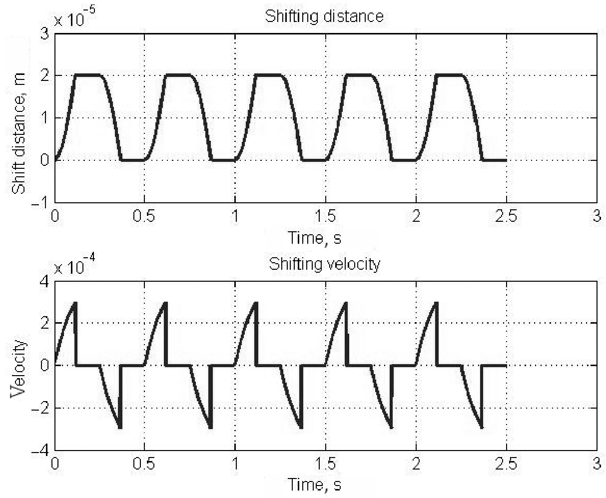

Fig. 5. Movement of the ball in a cavity; $q = 6 \times 10^{-16}$ [C].

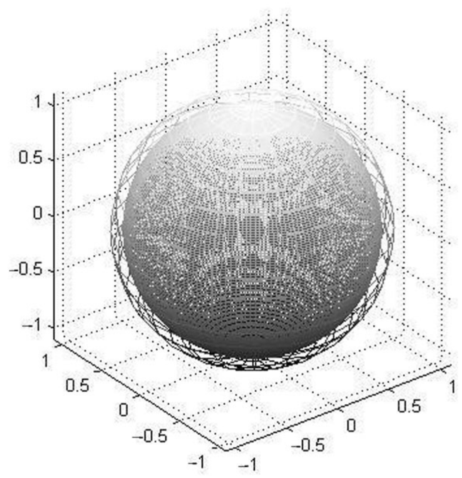

Fig. 6. Schematic view of a bichromal ball in a cell.

The mass m of the particle is

$$m = \rho \cdot V = \frac{4}{3} \cdot \pi \cdot \rho \cdot r^3. \tag{10}$$

Although in real systems the magnitude of the electric field varies across the thickness of the material due to the difference of the permittivity of silicone, carrier liquid and ball material, we accept a simplification:

$$E = \frac{U}{s}. \tag{11}$$

The resulting differential equation is

$$\frac{d^2 y}{dt^2} + \frac{9 \cdot \eta}{2 \cdot \rho \cdot r^2} \cdot \frac{dy}{dt} - \frac{3 \cdot U \cdot q}{4 \cdot \pi \cdot s \cdot \rho \cdot r^3}. \tag{12}$$

The simulated trajectory of the ball governed by Eq. (8) is shown in Fig. 5. The initial conditions are $y(0) = y'(0) = 0$.

4.2. Rotation of the ball

Every polarized bichromal ball is embedded into a cell filled with a dielectric fluid. The volume of the cell is α^2 times larger than the volume of the ball. An illustration corresponding to a ball "sitting" inside a cell is shown in Fig. 6.

The rotation of the ball inside the cell cavity as a result of the outer electrical field can be described through the balance relation

$$M_J + M_S + M_E = 0, \tag{13}$$

where M_J is the inertial torque:

$$M_J = -J \frac{d^2}{dt^2}, \tag{14}$$

J is the moment of inertia of the ball:

$$J = \frac{2}{5} m \cdot r^2 = \frac{8}{15} \pi \cdot \rho \cdot r^5, \tag{15}$$

M_S is the viscous torque:

$$M_S = -2\pi \cdot \eta \cdot r^4 \cdot \frac{d}{dt} \int (\sin\alpha)^3 d\alpha = -\frac{8}{3} \pi \cdot \eta \cdot r^4 \frac{d}{dt}, \tag{16}$$

and M_E is the electrostatic torque:

$$M_E = 2 r_D \cdot q_D \cdot E \cdot \sin\varphi. \tag{17}$$

The resulting differential equation describes the rotation of the ball:

$$-\frac{8}{15} \pi \cdot \rho \cdot r^5 \frac{d^2 \varphi}{dt^2} - \frac{8}{3} \pi \cdot \eta \cdot r^4 \frac{d\varphi}{dt} + \frac{r \cdot q_D \cdot U \cdot \sin\varphi}{S} = 0. \tag{18}$$

The initial conditions are $\varphi(0) = \varphi'(0) = 0$. The magnitude of rotation depends on several parameters. Figure 7 shows a sample movement of a ball free of the cavity.

If the ball is in the cavity, physical boundaries are applied. The changes of the angle and velocity of ball rotation corresponding to the situation above (Fig. 5) with different electrical parameters of the ball are shown in Figs 8, 9, and 10. In Fig. 8 we see that in the case of small dipole charge the ball performs some incomplete rotation cycles and stops in fixed angle.

In Fig. 9 the ball has a nearly optimal dipole charge; other parameters remain unchanged. We see that in this case the ball rotation intends towards some complete rotation cycle. If control voltage changes, the ball will rotate nearly 180° and expose the right, black or white size to the observer. The display works as expected.

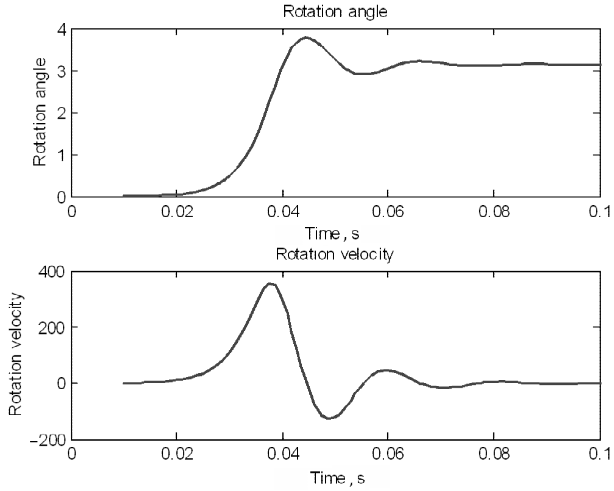

Fig. 7. Sample plot of ball rotation.

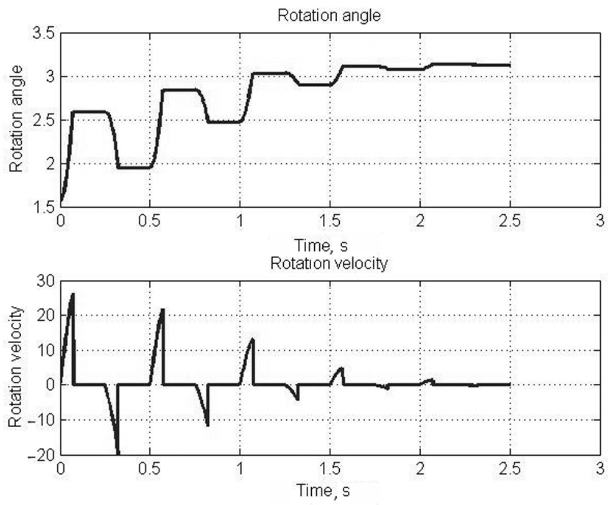

Fig. 8. Rotation angle and velocity of the ball in a cavity ($q_D = 1 \times 10^{-18}$ [C]).

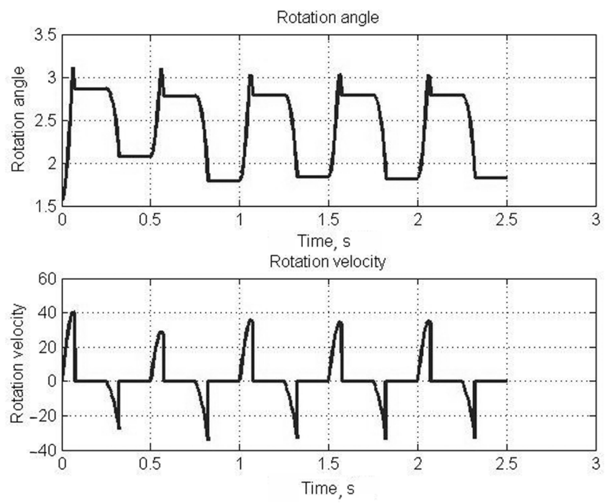

Fig. 9. Rotation angle and velocity of the ball in a cavity ($q_D = 2 \times 10^{-18}$ [C]).

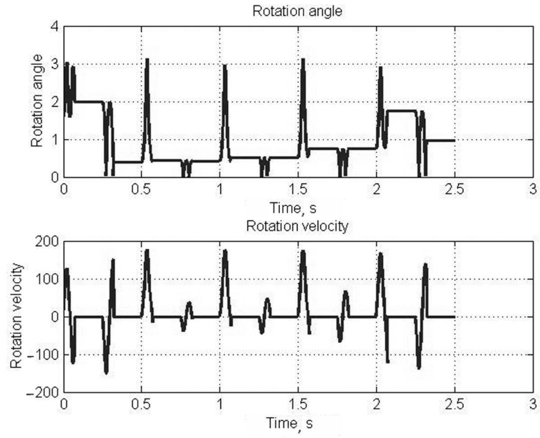

Fig. 10. Rotation angle and velocity of the ball in a cavity ($q_D = 1 \times 10^{-16}$ [C]).

In Fig. 10 we see that when the dipole charge is too large, over some critical value, then the uniform rotation cycles of the ball are lost and the ball will perform random rotations and stop in unpredictable states.

5. CONCLUSIONS

During numerous laboratory experiments, we discovered an unstable behaviour of the experimental display for different dipole and monopole charges of balls. In some cases the display worked as expected, but in some cases we noticed that the particles acquired random states, not the white and black as presumed, and in other cases the display stopped working at all after some time. Note that in the case of an unpropitious combination of physical parameters the rotation of the ball is strictly limited and the final rotation angle is highly undetermined. These situations correspond to the simulations presented in Figs 8 and 10.

We can conclude that the mathematical model presented in this paper corresponds to a proper operation of the display. In our experiments the dipole charge could be changed by changing the polarization parameters and the monopolar charge can be simply controlled using nonpolar surfactants dissolved in the carrier liquid. A proper operation of the display can be achieved only using the strictly predetermined combination of physical characteristics of the particles.

ACKNOWLEDGEMENT

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***Twisting-ball*'i tüüpi kuvari matemaatiline modelleerimine**

Jüri Liiv, Aleksei Mashirin, Toomas Tenno ja Peep Miidla

Polüvinülideendifluoriid (PVDF) sobib oma füüsikaliste omaduste poolest *twisting-ball*'i tüüpi displei (e-paber) valmistamiseks. Artiklis on matemaatiliselt modelleeritud sellest materjalist valmistatud aktiivelemendi (vedelikuga täidetud õõnsuses paiknev polariseeritud ja kahevärviline kerake) käitumine juhtpinge muutumisel. On koostatud osakese liikumist kirjeldavad diferentsiaalvõrrandid ja lahendatud need Runge-Kutta meetodil, kasutades programmi MATLAB. Tulemused näitavad, et displei korrektseks funktsioneerimiseks on vajalik osakeste füüsikaliste parameetrite kindel kombinatsioon.