Vertical crustal movements in Estonia determined from precise levellings and observations of the level of Lake Peipsi

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Abstract. The aim of this study was to evaluate vertical velocities of the benchmarks and their change over time based on the four precise levellings of the Estonian levelling network from 1933 to 2011, with the mean epochs being 1936.7, 1961.2, 1982.1 and 2006.9. The vertical velocities of the benchmarks were estimated using two mathematical models. Both models gave similar results for almost all levelling combinations. Significant discrepancies between the velocities from two models were found only in two combinations where levelling loops' closing time was long compared to the time between the mean epochs of levellings. From the analysis of post-adjustment variances of unit weight and the ANOVA test, a significant change in the benchmark velocities between mean epochs of the levellings was detected. However, due to correlation between the second and third levellings it remained unresolved whether the velocity change was a real change or fortuitous when relying only on this correlation. The detected velocity change could also be explained by the levelling error. Iterated variance component estimation assigned most of the error to the first levelling. In addition, level records of Lake Peipsi from 1921 to 2006 were used for the first time to calculate lake tilt between water gauges. Velocities of the benchmarks from the combination of the last three levellings and water gauges of Lake Peipsi were used to compile the map of the vertical crustal movements (EST2015LU). The main feature of the compiled map was the SE–NW directional postglacial land uplift. However, compared to earlier maps for the region, our isolines declined more in the W–E direction, due to the larger influence of the fourth levelling and velocities from lake tilts. Overall fit of the compiled map with the velocities of continuously operating Global Navigation Satellite System reference stations and coastal tide gauges was ±0.4 to ±0.5 mm yr^{-1}.

Key words: vertical crustal movement, levelling, lake tilts, land uplift.

Abbreviations: BM – benchmark, CLN – common levelling network, CORS – continuously operating GNSS reference station, GNSS – Global Navigation Satellite System, LSQ – least squares, RMS – root mean square, SA – satellite altimetry, TG – tide gauge, VCM – vertical crustal movement, VV – vertical velocity, WG – water gauge.

INTRODUCTION

Repeated levelling has remained one of the most precise methods for the determination of vertical movements of the Earth's crust. Many different methods for the calculation of vertical crustal movements (VCM) from repeated levelling data have been developed (Holdahl 1978; Carrera & Vaníček 1986; Hein 1986). Most of the methods were developed between the 1960s and 1980s, when a number of countries finished repeated levellings of their levelling networks. Until now, most of the Estonian levelling network has been levelled at least four times. This provides an excellent opportunity to estimate changes in VCMs over time using different levelling combinations.

The first precise levelling covering all of Estonia was completed between 1933 and 1943. In 1943 the network consisted of six loops on the mainland and one loop on the Island of Saaremaa, altogether approximately 2000 km of levelling lines and 1300 benchmarks (BMs); 23 nodal points of the network were deep-seated 'fundamental benchmarks'. The second and third levellings were carried out during 1948–1969 and 1970–1996, respectively. The fourth levelling was performed between 2001 and 2012 using digital levelling methodology and shorter sight lengths. More details about the repeated levellings in Estonia are provided by Kall et al. (2014).

The first maps of VCMs of inland Estonia were compiled from the results of the first two levellings (Zhelnin 1958, 1960, 1964, 1966; Randjärv 1968; Vallner & Zhelnin 1975; Vallner 1978). Values of VCMs in Estonia were previously known only from coastal tide gauge (TG) observations (e.g., Witting 1922). Results from the levellings confirmed that a general trend of land uplift in Estonia occurred in a SE–NW direction and correlated well with VCMs for southern Finland calculated from repeated levellings by Kääriäinen (1953, 1963). When the third levelling data for Estonia became available, the values of vertical velocities (VVs) of the BMs were re-estimated (Vallner et al. 1988; Randjärv 1993). It was concluded that in addition to general postglacial land uplift, differentiated block movements took place. The accuracy of the velocities was estimated to be on average between ±0.2 and ±0.4 mm yr^{-1} (Vallner & Zhelnin 1975; Vallner et al. 1988; Randjärv 1993).

Water level observations on the coast of Lake Peipsi (Fig. 1), Estonia's largest (3555 km^2) and the fifth largest lake in Europe, have been conducted by the Estonian Weather Service since 1921. The main purpose of the observations is water management: regulation of outflows, amelioration, building of hydraulic structures, etc. In the present study, water level observations were used for the first time for geodetic purposes to calculate land uplift.

The aim of this study was to estimate VCMs from different levelling combinations as well as from lake level recordings and to evaluate VV changes over time. This paper is organized as follows. The first two sections provide an overview of the levelling data used and uncertainties regarding them. An overview of the calculations used to estimate VVs of the BMs from levelling data is given. The differences between the obtained solutions are presented and analysed. The next two sections discuss the change in VVs over time and evaluate levelling errors based on variance component estimation. Next, an overview of VV calculation based on lake level observations is provided. The last two sections are devoted to the compilation and evaluation of the VCM models for Estonia.

LEVELLING OBSERVATIONS AND STATIC ADJUSTMENT OF THE NETWORK

To determine temporal changes in VVs of the BMs, a common levelling network (CLN) based on the levelling lines and BMs common to all four levellings (1933–1943, 1948–1969, 1970–1996 and 2001–2012; mean epochs 1936.7, 1961.2, 1982.1 and 2006.9, respectively) was created (Fig. 1).

Compared to the so-called maximum network, where all levelling lines of a network can be used, including lines levelled just one time, the CLN has some advantages: (i) estimation of the VVs from two different mathematical models (Eqs (4) and (5)), (ii) estimation of the change in the VVs of BMs over time (Eqs (9)–(11)), (iii) determination of the between-epoch correlation of the repeated levellings (Eqs (12)–(20)). Levelling observations without gravity correction were used in our study. However, the small systematic error introduced by ignoring gravity anomalies will be cancelled from VV calculations when height differences are differentiated in the completely re-levelled network and re-levelled segments follow close paths (Brown & Oliver 1976; Vaníček 1976; Holdahl 1978; Vaníček et al. 1980; Carrera et al. 1991). This was the case in our CLN.

In order to be a 'true' CLN, height differences between BMs and the nodal points of the network should be common to all levellings (Mäkinen & Saaranen 1998). It was necessary sometimes to sum adjacent height differences from different observation epochs to achieve closed network loops. In such cases the observation epoch of the summarized height difference was obtained as the weighted average where the section length was used as weight. The effect of epoch averages on adjustment results was negligible, since variation between the observation epochs of the adjacent height differences was small compared to the time-lag between the repeated levelling campaigns.

For each levelling line in the network, comparative graphs of the relative cumulative VVs of the BMs along the levelling line and a time series of section height differences were composed. Based on the graphs, as many as possible anomalously behaving BMs (peaks, slope or sign changes of the VVs on the graph (Giménez et al. 2000)) were removed from the network. Generally, such BMs had been mounted in unstable buildings, on unstable ground, or were located in areas of known local VV anomalies (cities of Tallinn, Pärnu and Tartu, and East Estonia, Fig. 1). Velocity anomalies in those areas have been associated with fluctuations in groundwater level, intensive groundwater consumption or oil shale mining (Lutsar et al. 1973; Mets et al. 2000; Kall & Torim 2003; Rüdja 2004; Kalm 2007).

To determine the weights w of the height differences, misclosures of the levelling loops c were used (Table 1).

A priori levelling standard error estimates were calculated for every levelling campaign using the equation

$$\tau = \sqrt{\frac{1}{n} \cdot \sum_{i=1}^{n} \frac{c_i^2}{P_i}}, \tag{1}$$

where n is the number of loops, c_i is the misclosure of the levelling loop (mm) and P_i is the perimeter of the levelling loop (km). *A priori* levelling standard error estimates τ based on the loops' misclosures in Table 1 are presented in Table 5.

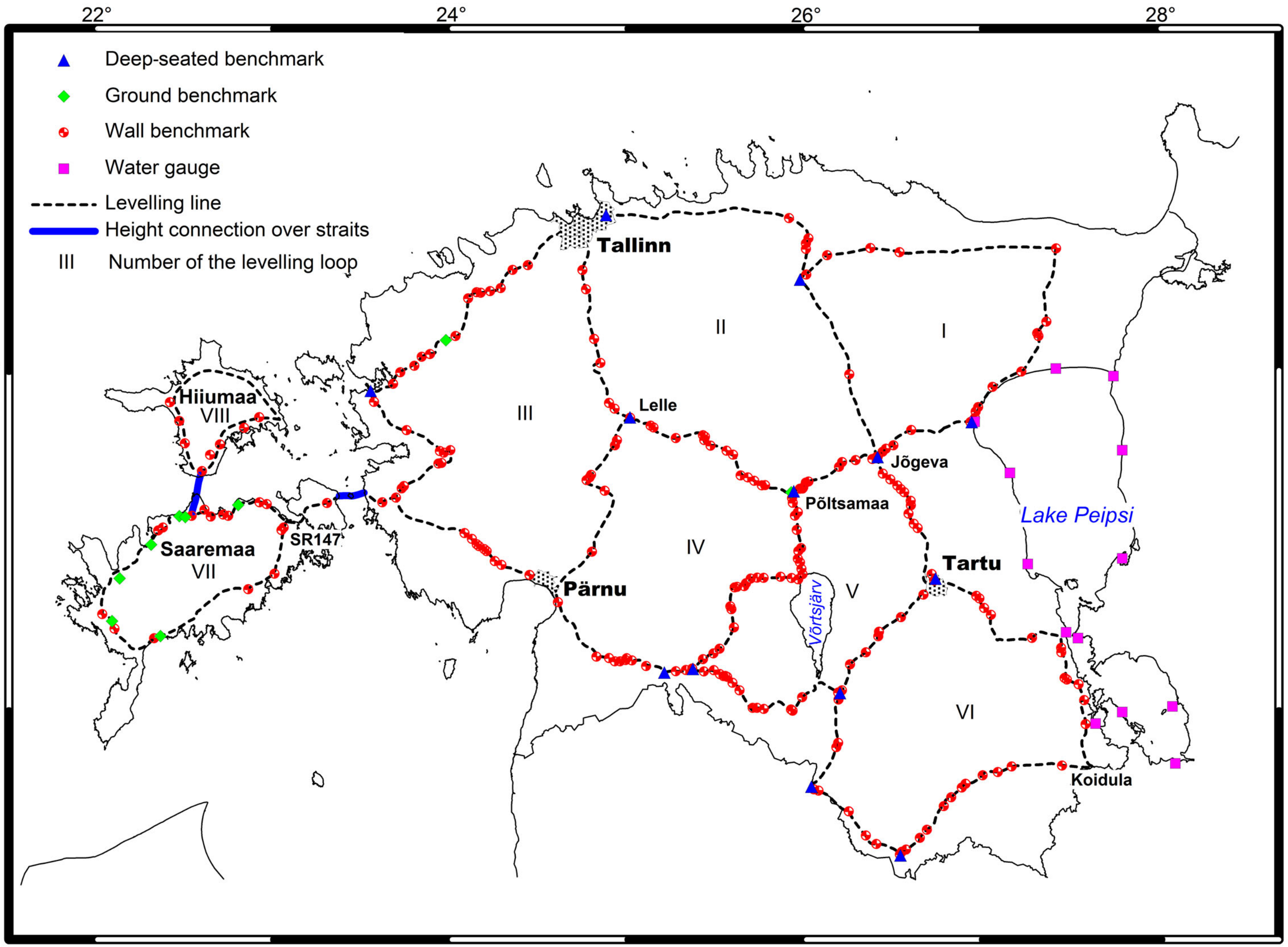

Fig. 1. The common levelling network based on the four precise levellings of Estonia with the mean epochs 1936.7, 1961.2, 1982.1 and 2006.9 and water gauges on the coast of Lake Peipsi used in the current study. Since the network of the first levelling campaign (1933–1943) did not extend to the Island of Hiiumaa, the loop there was based on the common benchmarks (BMs) of the last three levellings. The total length of the levelling lines was 2190 km (2000 km in the first levelling), the number of BMs was 249 (241) and the number of sections was 255 (246). The number of WGs was 13.

Table 1. Misclosures of the levelling loops of the common levelling network. A loop was not established on the Island of Hiiumaa (loop VIII, Fig. 1) during the first levelling campaign

Levelling loop	Loop perimeter P (km)	Loop misclosure c (mm)/max time difference of the closing of the loop (years)			
		Levelling campaign			
		1	2	3	4
I	336.60	−7.14/1	−6.47/13	12.92/18	3.97/0
II	400.94	−0.16/6	−8.07/21	−8.90/22	2.66/7
III	455.41	−17.07/3	−11.42/25	*61.55/16	−2.06/1
IV	396.73	13.28/2	6.29/13	−28.15/12	−4.91/1
V	336.96	0.15/3	1.06/14	30.30/10	5.87/1
VI	387.75	2.20/4	15.00/3	−18.00/7	−3.54/3
VII	210.35	**50.70/1	1.02/0	−0.03/0	1.63/0
VIII	132.89	–	4.72/0	−26.52/0	1.18/2

* The large closing error might be related to the contribution of land uplift, considering the loop's closing time and the fact that this loop is located in the area of the highest rate of land uplift in Estonia. For example, after implementing land uplift correction, the closing error of this loop was reduced down to ~32 mm, depending on the VVs used for corrections.

** The levelling loop on the Island of Saaremaa (loop VII) was levelled with third order precision (Vaníček et al. 1980). Therefore, the misclosure of this loop is relatively bigger in terms of Vaniček et al. (1980) compared to other loops within the same levelling campaign.

Variances of height differences between BMs were calculated by using the formula

$$m_i^2 = \tau_i^2 L, \tag{2}$$

where L is the length of the section (km).

Variances of the observations were used to compose the covariance matrix $\mathbf{\Sigma}$ of the observations. Weight matrix $\mathbf{W}$ is related to observations error vector $\mathbf{e}$, covariance matrix of observations $\mathbf{\Sigma}$ and cofactor matrix $\mathbf{Q}$ of observations as follows (Ghilani & Wolf 2006, pp. 159–172):

$$\mathbf{\Sigma} = E(\mathbf{ee}^{\mathrm{T}}) = \sigma_0^2 \mathbf{W}^{-1} = \sigma_0^2 \mathbf{Q}, \tag{3}$$

where σ_0^2 is the *a priori* variance of unit weight, 1.

Least squares (LSQ) adjustment (with minimum constraints) of the observations of each levelling campaign was then performed separately and *a posteriori* variances of unit weight was estimated using a χ^2-test. Post-adjustment variance of unit weight estimations did not differ significantly from the *a priori* value of 1 $(\alpha = 0.05)$. Consequently, the weights used in the adjustments were internally correct as far as the individual levelling campaign adjustments were concerned. In the case of the kinematic adjustment of two or more levelling combinations, *a posteriori* variance of unit weight depends also on the relationships between the weights of the observations of separate levelling campaigns.

MATHEMATICAL MODELS OF THE KINEMATIC ADJUSTMENT OF THE LEVELLING NETWORK

According to the Gauss–Markov model $\mathbf{Y} = \mathbf{AX} + \mathbf{e}$, an observation equation that relates observations (height differences and corresponding levelling epochs) with the heights and VVs of BMs ('heights included' model) in the case of four levellings, is

$$\begin{pmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \mathbf{y}_3 \\ \mathbf{y}_4 \end{pmatrix} = \begin{pmatrix} \mathbf{A}_1 & \mathbf{T}_1\mathbf{A}_1 \\ \mathbf{A}_2 & \mathbf{T}_2\mathbf{A}_2 \\ \mathbf{A}_3 & \mathbf{T}_3\mathbf{A}_3 \\ \mathbf{A}_4 & \mathbf{T}_4\mathbf{A}_4 \end{pmatrix} \begin{pmatrix} \mathbf{H} \\ \mathbf{v} \end{pmatrix} + \begin{pmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \\ \mathbf{e}_3 \\ \mathbf{e}_4 \end{pmatrix}, \tag{4}$$

where $\mathbf{A}$ is the design matrix of the levelling network; $\mathbf{H}$ is the vector of the unknown heights of the BMs at the arbitrary chosen reference epoch t_0; $\mathbf{v}$ is the vector of the unknown VVs of the BMs; $\mathbf{y}$ is the vector of the levelling observations (height differences); $\mathbf{e}$ is the error vector of the levelling observations and $\mathbf{T} = \mathrm{diag}(t_i - t_0, \ldots, t_m - t_0)$ is the diagonal matrix of the levelling epochs.

All four levellings in the CLN had the same design matrix, i.e. $\mathbf{A}_1 = \mathbf{A}_2 = \mathbf{A}_3 = \mathbf{A}_4 = \mathbf{A}$. In such a network it is possible to eliminate unnecessary parameters by calculating the differences between the observed height differences of the same BMs. As a result, a 'heights eliminated' model, where only VVs are parameterized, is obtained (Mäkinen & Saaranen 1998; Mäkinen 2002):

$$\begin{pmatrix} (\mathbf{y}_2 - \mathbf{y}_1) \\ (\mathbf{y}_3 - \mathbf{y}_2) \\ (\mathbf{y}_4 - \mathbf{y}_3) \end{pmatrix} = \begin{pmatrix} (\mathbf{T}_2 - \mathbf{T}_1)\mathbf{A} \\ (\mathbf{T}_3 - \mathbf{T}_2)\mathbf{A} \\ (\mathbf{T}_4 - \mathbf{T}_3)\mathbf{A} \end{pmatrix} \mathbf{v} + \begin{pmatrix} (\mathbf{e}_2 - \mathbf{e}_1) \\ (\mathbf{e}_3 - \mathbf{e}_2) \\ (\mathbf{e}_4 - \mathbf{e}_3) \end{pmatrix}. \quad (5)$$

By differentiating observations, the new observations, which are basically differences of VVs between the BMs, are obtained. They are correlated through the second and third levelling datasets:

$$\begin{pmatrix} (\mathbf{e}_2 - \mathbf{e}_1) \\ (\mathbf{e}_3 - \mathbf{e}_2) \\ (\mathbf{e}_4 - \mathbf{e}_3) \end{pmatrix} = \sigma_0^2 \begin{pmatrix} \mathbf{Q}_1 + \mathbf{Q}_2 & -\mathbf{Q}_2 & 0 \\ -\mathbf{Q}_2 & \mathbf{Q}_2 + \mathbf{Q}_3 & -\mathbf{Q}_3 \\ 0 & -\mathbf{Q}_3 & \mathbf{Q}_3 + \mathbf{Q}_4 \end{pmatrix}. \quad (6)$$

Gauss–Markov estimates of the VVs from the 'heights eliminated' model are the best linear unbiased estimators. However, compared to the 'heights included' model it provides only linear unbiased estimators, since the velocity information contained in the loops' misclosures is lost when levelling observation differences are formed (Cross et al. 1987; Mäkinen 2002). Different estimates for the VVs from the 'heights included' and 'heights eliminated' models might therefore be expected. It is theoretically possible that if loop misclosure contains some systematic errors, they remain in the parameters of the 'heights included' model. However, such systematic errors are eliminated when applying the 'heights eliminated' model. Therefore, systematic errors can also be the reason for the different VV estimates. Mäkinen & Saaranen (1998) have shown that VVs from the two different models do not differ in practice. Only the error estimates of the parameters can be different.

Results of the common levelling network adjustment

As shown in Table 2, it was possible to form eleven combinations of the four levellings to calculate VVs of the CLN BMs (Fig. 1). In addition, VVs were calculated using both the 'heights included' and 'heights eliminated' models. Since details of all adjustments would take

Table 2. The main statistics of the kinematic adjustment of the common levelling network, grouped by the adjustment model and levelling combinations: degrees of freedom *df*, variances of unit weight S_0^2 and the corresponding *p*-values of the null-hypothesis $S_0^2 = \sigma_0^2$; comparison of variances of unit weight S_0^2 and mean standard errors μ of the adjusted vertical velocity differences from the 'heights included' and 'heights eliminated' models, with *p*-values for the null-hypotheses $S_{0_1}^2 / S_{0_2}^2 = 1$ and $\mu_1^2 / \mu_2^2 = 1$. Null hypotheses were rejected if $p < 0.05$

Statistic		Levelling combination										
		1–2–3–4	1–2–3	1–2–4	1–3–4	2–3–4	1–2	1–3	1–4	2–3	2–4	3–4
'heights included'	*df*	504	258	258	258	269	12	12	12	14	14	14
	$S_{0_1}^2$	10.078	7.959	16.364	4.768	3.192	1.614	0.905	1.222	0.672	1.227	1.055
	p-value	0.000	0.000	0.000	0.000	0.000	0.080	0.541	0.198	0.804	0.192	0.322
	μ_1	0.312	1.424	0.400	0.459	0.196	0.939	0.538	0.245	4.774	0.123	0.711
'heights eliminated'	*df*	498	252	252	252	262	6	6	6	7	7	7
	$S_{0_2}^2$	10.185	8.108	16.726	4.848	3.248	1.025	1.060	1.129	0.824	1.345	0.799
	p-value	0.000	0.000	0.000	0.000	0.000	0.292	0.384	0.238	0.567	0.224	0.471
	μ_2	0.314	1.549	0.405	0.463	0.198	0.853	0.585	0.236	5.310	0.128	0.622
$F = S_{0_1}^2 / S_{0_2}^2$		1.011	1.019	1.022	1.017	1.018	1.575	1.171	1.083	1.226	1.097	1.321
p-value		0.453	0.441	0.431	0.447	0.443	0.323	0.383	0.499	0.352	0.419	0.384
$F = \mu_1^2 / \mu_2^2$		1.006	1.088	1.012	1.009	1.010	1.101	1.088	1.040	1.112	1.048	1.144
p-value		0.473	0.251	0.463	0.471	0.469	0.493	0.422	0.519	0.408	0.446	0.460

up much space, only discrepancies in VVs between the two models and two error estimates of the kinematic adjustments in the form of (i) *a posteriori* variance of unit weight S_0^2 and (ii) mean standard error of the adjusted VV differences μ are presented. Variance of unit weight S_0^2 was calculated by the formula

$$S_0^2 = \frac{\mathbf{e}^{\mathrm{T}}\mathbf{W}\mathbf{e}}{df}, \tag{7}$$

where $\mathbf{W}$ is the weight matrix, df is the degrees of freedom of the network and $\mathbf{e}$ is the vector of the residuals obtained from the adjustment. The mean standard error of the adjusted VV differences, μ, was obtained by the formula

$$\mu = \sqrt{\frac{tr[\mathbf{Q}_{\mathbf{xx}}]}{n}} \cdot S_0, \tag{8}$$

where $\mathbf{Q}_{\mathbf{xx}}$ is the parameter cofactor matrix and n is the number of velocity components.

A χ^2-test was performed to test the closeness of S_0^2 to the *a priori* value $\sigma_0^2 = 1$. The comparison of S_0^2 and μ from the 'heights included' and 'heights eliminated' models was performed using an *F*-test. The results of statistical evaluation of the post-adjustment statistics are presented in Table 2.

Dependence of the standard error of the vertical velocity differences on the levelling combination used

According to the χ^2-test, *a posteriori* variances of unit weight (S_0^2) from models containing three and four levelling combinations differed significantly from an *a priori* value of 1 ($p < 0.05$), independently of the model used (Table 2). The fact that the S_0^2 of two levelling combinations did not differ from unity but all three and four levelling combinations did indicated that the VVs of the BMs between levelling periods were uneven, observations contained errors or weight matrices of the observations did not fit together.

The mean standard errors (μ) of the calculated VV differences were strongly influenced by including the fourth levelling into the adjustment. This was probably related to the weights of the fourth levelling being approximately two times higher than the others. Additionally, leaving out the middle levelling from a combination of three did not influence error estimates significantly (e.g., 2–3–4 compared with 2–4). Mäkinen & Saaranen (1998) explained this effect using simple linear regression as an example, where leaving out an observation in the middle of abscissa values does not significantly influence the value of the slope.

The standard error of the VV differences was also influenced by the time period between levellings. A longer time period between levelling central epochs contributed to smaller standard errors (e.g., combinations 1–3 compared with 1–2 or 2–3, 2–4 compared with 2–3 or 3–4). The large standard error in the combination 2–3 was related to the very short time period (1 year) between the repeated hydrostatic levellings connecting the Islands of Saaremaa and Hiiumaa (Fig. 1), plus the large discrepancy (10.2 mm) between the height differences. The adjustment of the same levelling combination for the mainland part of the CLN plus loop on the Island of Saaremaa gave an approximately four times smaller mean standard error. When adjustment was performed only with the mainland part of the CLN, the mean standard error of the VV differences reduced further two times. This indicates that the height connections of the Island of Saaremaa with the mainland or the levelling loop on Saaremaa also affect error estimates in the levelling combination 2–3.

Dependence of the standard error of the vertical velocity differences on the type of the model used

Differences of S_0^2 between the 'heights included' and 'heights eliminated' models can be related to (i) differentiating the observations in loops with the same loops' misclosure signs and (ii) solution of the 'heights eliminated' model of two levelling combinations being independent of weights (Mäkinen & Saaranen 1998). However, this was not the case in our CLN, where differences of S_0^2 between the 'heights included' and 'heights eliminated' models were insignificant ($p < 0.05$) in all levelling combinations (Table 2). Differences between mean standard error μ from the 'heights included' and 'heights eliminated' models were also insignificant. Differences between the ratios $S_{0_1}^2 / S_{0_2}^2$ of different levelling combinations were most likely caused by the differences between the ratios of observation weights for the levellings involved. The solution for two levelling combinations in the 'heights eliminated' model is not influenced by this weight ratio, whereas the solution from the 'heights included' model is the best linear unbiased estimator only when the ratio of weights from the two levellings is correct (Mäkinen & Saaranen 1998). This difference, however, only concerns the error estimates; the VVs from the two models were identical for the majority of levelling combinations.

Dependence of vertical velocity estimates on the model used

As mentioned above, for most levelling combinations no significant differences existed between VVs between the ‘heights included’ and ‘heights eliminated’ models. Indeed, velocity estimates from the two solutions should coincide if the levelling loops were closed in a short time period and the reference epoch for the adjustment t_0 was chosen so that the levellings were performed symmetrically with respect to it (Cross et al. 1987). Significant VV differences between the ‘heights included’ and ‘heights eliminated’ models were obtained only for the combinations 1–2–3 and 1–2. Large VV differences (although not statistically significant) were also obtained from the combinations 2–3 and 3–4. The transformation of the observations to the mean for each levelling period epoch eliminated the differences between the VVs of the ‘heights included’ and ‘heights eliminated’ models, showing that differences were caused by a longer time period of the closing of the loops during the second and third levellings compared to the first and fourth levellings (Table 1).

CHANGE IN THE VERTICAL VELOCITIES OVER TIME

Based on the differences between VVs from the different levelling combinations, the sum of the squared observation residuals and degrees of freedom of the kinematic adjustment, it is possible to evaluate the significance of the VV changes between mean levelling epochs using an ANOVA test (Kakkuri & Vermeer 1985; Mäkinen & Saaranen 1998).

To reckon with a correlation between the VVs (for example, velocities from the combinations 1–2 and 2–3 are correlated through the observations of the second levelling), the ‘heights eliminated’ model Eq. (5) was rewritten so that it was possible to obtain three VVs for each BM from a single adjustment:

$$\begin{pmatrix} (\mathbf{y}_2 - \mathbf{y}_1) \\ (\mathbf{y}_3 - \mathbf{y}_2) \\ (\mathbf{y}_4 - \mathbf{y}_3) \end{pmatrix} = \begin{pmatrix} (\mathbf{T}_2 - \mathbf{T}_1)\mathbf{A} \\ (\mathbf{T}_3 - \mathbf{T}_2)\mathbf{A} \\ (\mathbf{T}_4 - \mathbf{T}_3)\mathbf{A} \end{pmatrix} \begin{pmatrix} \mathbf{v}_{1-2} \\ \mathbf{v}_{2-3} \\ \mathbf{v}_{3-4} \end{pmatrix} + \begin{pmatrix} (\mathbf{e}_2 - \mathbf{e}_1) \\ (\mathbf{e}_3 - \mathbf{e}_2) \\ (\mathbf{e}_4 - \mathbf{e}_3) \end{pmatrix}, \quad (9)$$

where $\mathbf{v}_{1-2}, \mathbf{v}_{2-3}, \mathbf{v}_{3-4}$ are the VVs of the BMs between the first and second, second and third, and third and fourth mean levelling epochs, respectively. The remaining terms were introduced earlier.

The model (Eq. (9)) established the reference for the evaluation of VV change between the mean levelling epochs where the correlation through the second and third levellings is automatically taken into account. In order to test the hypothesis that the VVs of the BMs were constant in time (i.e., $\mathbf{v}_{2-3} - \mathbf{v}_{1-2} = \mathbf{v}_{3-4} - \mathbf{v}_{2-3} = \mathbf{v}_{3-4} - \mathbf{v}_{1-2} = 0$), VV differences $(v_k - v_j)$ were found. The mean square of VV differences is

$$S^2_{dv_i} = \frac{\sum_1^n (v_k - v_j)^2}{n}, \quad (10)$$

where $j = 1–2,\ 2–3$ and $k = 2–3,\ 3–4$ are levelling combinations and n is the number of BMs. Based on the mean square values for both the VV differences from Eq. (10) and the residuals from Eq. (9) found in column 4 in Table 3, we obtained the F-statistic as the quotient between the two, as listed in column five. By comparing this value with the critical value F_{crit}, we found the p-values listed in column six.

The difference between the VVs from levellings 1–2 and 3–4 was not statistically significant, whereas VVs from the 1–2 and 2–3 levellings, as well as the 2–3 and 3–4 levellings, were significantly different ($p < 0.05$, Table 3). The results suggest that there was a significant change in VVs between the second and third levellings. The ANOVA confirmed no significant changes in VV between the levelling pairs 1–2 and 2–4, and 1–3 and 3–4. Similar ANOVA results were also obtained for the VVs from the adjustment of the whole CLN, i.e., height differences from the straits between the mainland and islands and loops on the islands did not influence the results of the analysis.

Even though the performed ANOVA test (Table 3) is invariant to the overall scaling of the *a priori* weights, it is sensitive to the ratios of the weights of separate levellings. In order to verify that the difference between the VVs $\mathbf{v}_{3-4}$ and $\mathbf{v}_{1-2}$ was not statistically significant, we did a new test using a model less sensitive to the *a priori* weight ratio. The model for the evaluation of

Table 3. The results of the ANOVA testing the hypothesis that differences of the vertical velocities (VVs) from Eq. (9) were zero, i.e., the VVs of the BMs were constant in time. The results are given only for the mainland part of the common levelling network

Source	Sum of squares	Degrees of freedom	Mean square	F-statistic	p-value
$\mathbf{v}_{2-3} - \mathbf{v}_{1-2}$	376.759	215	1.752		
Residual	8.073	15	0.538		
ANOVA				3.256	0.0060
$\mathbf{v}_{3-4} - \mathbf{v}_{2-3}$	295.796	215	1.376		
Residual	8.073	15	0.538		
ANOVA				2.556	0.0200
$\mathbf{v}_{3-4} - \mathbf{v}_{1-2}$	135.642	215	0.631		
Residual	8.073	15	0.538		
ANOVA				1.172	0.3800

VV differences $\mathbf{v}_{3-4} - \mathbf{v}_{1-2}$ was obtained by subtracting the first from the third row in Eq. (9) (Mäkinen & Saaranen 1998):

$$(\mathbf{T}_4 - \mathbf{T}_3)^{-1}(\mathbf{y}_4 - \mathbf{y}_3) - (\mathbf{T}_2 - \mathbf{T}_1)^{-1}(\mathbf{y}_2 - \mathbf{y}_1) = \mathbf{A}(\mathbf{v}_{3-4} - \mathbf{v}_{1-2}) + (\mathbf{T}_4 - \mathbf{T}_3)^{-1}(\mathbf{e}_4 - \mathbf{e}_3) - (\mathbf{T}_2 - \mathbf{T}_1)^{-1}(\mathbf{e}_2 - \mathbf{e}_1). \quad (11)$$

Based on the VV differences from the solution of Eq. (11), the hypothesis that $\mathbf{v}_{3-4} - \mathbf{v}_{1-2} = 0$ was tested using an ANOVA (see Table 4 for the results).

Though the alternative hypothesis $\mathbf{v}_{3-4} - \mathbf{v}_{1-2} \neq 0$ was not proven previously (see Table 3), in Table 4 the difference between the VVs $\mathbf{v}_{3-4}$ and $\mathbf{v}_{1-2}$ was also statistically significant ($p < 0.05$). The failure to reject the null hypothesis in Table 3 was probably related to the non-compatible weight ratios of the levellings used. Compared to the results in Table 3, the mean square of the residuals in Table 4 is now essentially an uncertainty estimate related to the misclosure of the VV differences on the left side of Eq. (11).

Although the ANOVA test for velocity change was performed before the outlier detection test, in our opinion, it did not affect results significantly. Change in the VVs of the BMs after removing outliers was small (-0.04 ± 0.14 mm yr^{-1} on average) compared to the VVs before the outlier test. This change can be considered insignificant relative to the standard deviation of the VVs from Eq. (9) (± 0.6 mm yr^{-1}). We also performed an ANOVA test in which detected outliers were not removed from the dataset but were weighted down using the 'Danish method' (Krarup et al. 1980). Conclusions about the change in the VVs of the BMs from this test were the same.

Table 4. Results of the ANOVA testing the hypothesis that differences of the vertical velocities $\mathbf{v}_{3-4} - \mathbf{v}_{1-2}$ from Eq. (11) equalled zero. The test included only the mainland part of the common levelling network

Source	Sum of squares	Degrees of freedom	Mean square	*F*-statistic	*p*-value
$\mathbf{v}_{3-4} - \mathbf{v}_{1-2}$	261.415	215	1.216		
Residual	0.474	5	0.095		
ANOVA				12.838	0.0040

As an example of changes in VVs, the profile of changes $\mathbf{v}_{2-3} - \mathbf{v}_{1-2}$ and $\mathbf{v}_{3-4} - \mathbf{v}_{2-3}$ in VVs solved from Eq. (9) for levelling loop VII in Saaremaa (Fig. 1) is presented in Fig. 2. The VV changes are relative to BM FR241 in Tallinn. The changes in VVs are far away from zero as was *a priori* hypothesized. Also, changes in VVs from different combinations are not close to each other. Opposite signs of the VV changes $\mathbf{v}_{2-3} - \mathbf{v}_{1-2}$ and $\mathbf{v}_{3-4} - \mathbf{v}_{2-3}$ suggest systematic levelling errors, most likely in the second or third levelling.

Correlations between the vertical velocities and velocity changes

Conclusions about the change in VVs with time (for our results see Tables 3 and 4) depend on the correlation between the levellings (Mäkinen & Saaranen 1998). The correlation can be a real relationship (for example, the same equipment was used in different levelling campaigns, thus, these levellings may share the same source of possible systematic errors) or just accidental, fortuitous. *A priori*, it was assumed that repeated levellings of the same levelling line have been performed independently from each other. In order to find out whether correlation existed between repeated levellings,

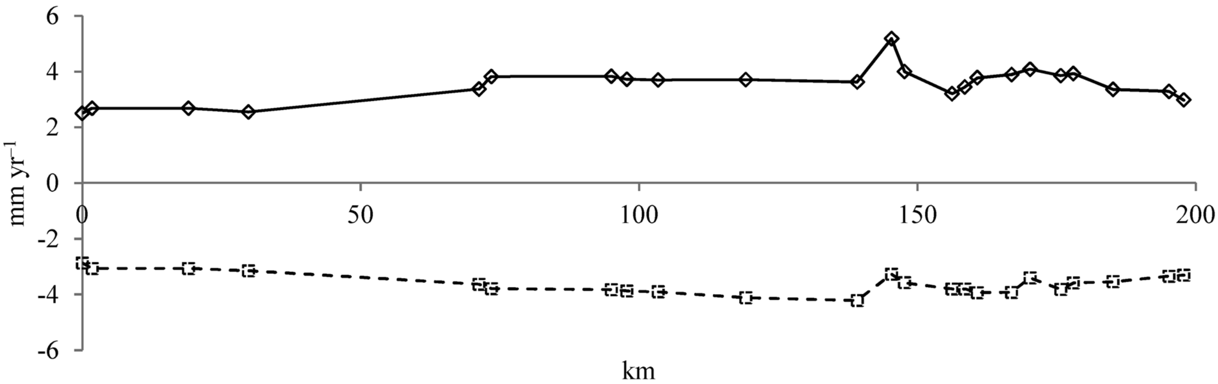

Fig. 2. Profile of changes $\mathbf{v}_{2-3} - \mathbf{v}_{1-2}$ (solid line) and $\mathbf{v}_{3-4} - \mathbf{v}_{2-3}$ (dashed line) in apparent VVs of the benchmarks (BMs) solved from Eq. (9) along levelling loop VII in Saaremaa starting from the wall BM SR147 (Fig. 1). Here $\mathbf{v}_{1-2}$, $\mathbf{v}_{2-3}$ and $\mathbf{v}_{3-4}$ are the average VVs between levellings 1 and 2, 2 and 3, and 3 and 4, respectively. Changes in the VVs are relative to BM FR241 in Tallinn.

multivariate analysis of the four levellings was carried out according to the algorithm by Mäkinen & Saaranen (1998).

(i) The height differences were transformed to a central epoch $\bar{t}_k$ of its levelling campaign $k = 1, 2, 3, 4$ using VVs $\tilde{\dot{\mathbf{h}}}$ from the kinematic adjustment of four levellings after the removal of the outliers and re-scaled weights:

$$\bar{\mathbf{y}}_k = \mathbf{y}_k + [(\bar{t}_k - t_0)\mathbf{I} - \mathbf{T}_k]\mathbf{A}\tilde{\dot{\mathbf{h}}}. \tag{12}$$

(ii) The same lengths for all levellings were used:

$$\mathbf{G} = \mathrm{diag}[\ldots,(d_1(i,j)\,d_2(i,j)\,d_3(i,j)\,d_4(i,j))^{\frac{1}{4}},\ldots]. \tag{13}$$

(iii) From the solution of the multivariate regression equation

$$(\bar{\mathbf{y}}_1\ \bar{\mathbf{y}}_2\ \bar{\mathbf{y}}_3\ \bar{\mathbf{y}}_4) = \mathbf{A}(\bar{\mathbf{h}}_1\ \bar{\mathbf{h}}_2\ \bar{\mathbf{h}}_3\ \bar{\mathbf{h}}_4) + (\mathbf{e}_1\ \mathbf{e}_2\ \mathbf{e}_3\ \mathbf{e}_4) \tag{14}$$

a between-epochs variance matrix $\mathbf{D}$ and the corresponding correlation matrix $\mathbf{\Pi}$ can be obtained.

(iv) $\mathbf{D}$ was estimated by the covariances between the residuals:

$$(\bar{\mathbf{h}}_1\ \bar{\mathbf{h}}_2\ \bar{\mathbf{h}}_3\ \bar{\mathbf{h}}_4) = (\mathbf{A}^{\mathrm{T}}\mathbf{G}^{-1}\mathbf{A})^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{G}^{-1}(\bar{\mathbf{y}}_1\ \bar{\mathbf{y}}_2\ \bar{\mathbf{y}}_3\ \bar{\mathbf{y}}_4), \tag{15}$$

$$(\mathbf{e}_1\ \mathbf{e}_2\ \mathbf{e}_3\ \mathbf{e}_4) = (\mathbf{I} - \mathbf{A}(\mathbf{A}^{\mathrm{T}}\mathbf{G}^{-1}\mathbf{A})^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{G}^{-1}(\bar{\mathbf{y}}_1\ \bar{\mathbf{y}}_2\ \bar{\mathbf{y}}_3\ \bar{\mathbf{y}}_4), \tag{16}$$

$$\mathbf{R}_0 = (\mathbf{e}_1\ \mathbf{e}_2\ \mathbf{e}_3\ \mathbf{e}_4)^{\mathrm{T}}\mathbf{G}^{-1}(\mathbf{e}_1\ \mathbf{e}_2\ \mathbf{e}_3\ \mathbf{e}_4), \tag{17}$$

$$\mathbf{D} = n^{-1}\mathbf{R}_0. \tag{18}$$

(v) To obtain the correlation matrix $\mathbf{\Pi}$, the vector of the standard deviations of the VVs $\mathbf{S}$ was calculated from the variance matrix $\mathbf{D}$:

$$\mathbf{S} = \sqrt{\mathrm{diag}(\mathbf{D})}, \tag{19}$$

and the correlation matrix equalled

$$\mathbf{\Pi} = \mathbf{S}^{-1}\mathbf{D}\mathbf{S}^{-1}. \tag{20}$$

In all formulas $\bar{\mathbf{y}}_k$ is the vector of the time-homogenized height differences of the levelling campaign $k = 1, 2, 3, 4$ at the central epoch $\bar{t}_k$ of the levelling k; t_0 is the reference epoch of the kinematic adjustment from which the VVs $\tilde{\dot{\mathbf{h}}}$ were obtained; $d_k(i,j)$ is the levelling distance between the BMs i and j; $\mathbf{h}_k$ is the vector of the BMs' heights of the levelling k at $\bar{t}_k$; $\mathbf{A}$ is the design matrix of the levellings; $\mathbf{e}_k$ is the error vector of the levelling k; n is the number of levelling loops.

Numerically:

$$\mathbf{D} = \begin{pmatrix} 1.925 & -0.037 & -0.430 & 0.043 \\ -0.037 & 0.330 & 0.590 & -0.020 \\ -0.430 & 0.590 & 2.203 & 0.110 \\ 0.043 & -0.020 & 0.110 & 0.030 \end{pmatrix} \mathrm{mm}^2\,\mathrm{km}^{-1} \tag{21}$$

and

$$\mathbf{\Pi} = \begin{pmatrix} 1 & -0.046 & -0.209 & 0.180 \\ -0.046 & 1 & 0.691 & -0.200 \\ -0.209 & 0.691 & 1 & 0.427 \\ 0.180 & -0.200 & 0.427 & 1 \end{pmatrix}. \tag{22}$$

The strong correlation between the second and the third levelling (0.691) was significant ($p < 0.05$). The second and third levellings were done using the same levelling methodology, and partly with the same team and equipment. Therefore, these levellings may share the same instrumental and methodological errors. The significant VV changes presented in Tables 3 and 4 can be related to the correlation between the second and the third levelling. At the same time it should remembered that the result of this test is approximate due to the time-homogenization.

ESTIMATION OF THE VARIANCE COMPONENTS

After the kinematic adjustment of the levelling combination 1–2–3–4, a variance of unit weight $S_0^2 = 10.078$ was obtained from the 'heights included' model (Table 2). The variance of unit weight reflected how the different levelling data and levelling error estimates fit together with the model with constant VVs. The larger the S_0^2, the poorer the fit. The detected VV change over time (Tables 3 and 4) could also be explained by levelling errors, if it is assumed that the levelling error is $S_0 = \sqrt{10.078} = 3.2$ larger than indicated by loop misclosures. Then the *a priori* levelling standard error estimates from Eq. (1) could be re-scaled uniformly by 3.2 times for all four levellings. But instead of the uniform rescaling, different variance factors S_0^2 were used for different levelling groups (k = 1,2,3,4):

$$\mathbf{\Sigma} = \mathrm{diag}\,(S_{0_1}^2\mathbf{W}_1^{-1}\quad S_{0_2}^2\mathbf{W}_1^{-1}\quad S_{0_3}^2\mathbf{W}_3^{-1}\quad S_{0_4}^2\mathbf{W}_4^{-1}), \tag{23}$$

where $\mathbf{W}_k$ is the weight matrix of the levelling group k (k = 1,2,3,4) and $\mathbf{\Sigma}$ is the covariance matrix of the observations.

There are many different methods for estimating $S_{0_k}^2$. The best known are the Helmert (Helmert 1872; Welsch 1978), Bique (Koch 1978, 2010; Welsch 1984), Minque (Rao 1971), Förstner (Förstner 1979) and IAUE (Lucas 1985) methods. An overview of different variance component estimation techniques can also be found in Amiri-Simkooei (2007) and Bähr et al. (2007).

Before the evaluation of the variance components, outliers were removed from the dataset using a 'data snooping' method (Baarda 1968; also described in Kall et al. 2014). The observation was rejected from the dataset if its standardized residual was larger than $S_0 \times 3.29$ $(\alpha = 0.001)$. The method is based on the assumption that there is only one outlier in the set of the observations. Therefore only one observation with the biggest standardized residual was removed from the set, after what adjustment was repeated. This was done iteratively until no outliers were detected. Altogether 25 iterations were performed. Most of the detected outliers belonged to the first levelling (altogether 17 outliers from 24). Four outliers belonged to the second levelling, two outliers to the third and one outlier to the fourth levelling. The outliers greatly influenced the *a posteriori* variance of unit weight, as was also brought out by Kall et al. (2014). From first to 25th iteration S_0^2 decreased from 10.08 to 1.79, i.e. about 6 times.

After removing the outliers from the dataset the variance component estimation of the four levelling campaigns of the CLN was performed using the Helmert, Bique and Förstner methods. After convergence to unity, the methods led to identical variance factors. The only difference was in the number of the iterations necessary (Helmert and Bique = 14 iterations, Förstner = 40 iterations). Variance factors and re-scaled levelling standard errors obtained after the variance component estimation are presented in Table 5.

The obtained variance factors (Table 5) were not homogeneous when compared to each other or with respect to the *a priori* levelling standard errors. Neither were the variance factors uniform with respect to the levelling standard errors obtained from Eq. (21) (diagonal elements of the variance matrix 1.387^2, 0.574^2, 1.484^2 and 0.173^2). These could serve for the best estimates of the levelling standard errors, adjusting every levelling campaign separately and considering only the interior fit of the height differences. The methodology for the estimation of the variance components in a constant VV model does not only consider the interior fit of the height differences but also their location in the time scale (Mäkinen & Saaranen 1998).

Table 5. Results of the variance component estimation using the Helmert, Bique and Förstner methods. All methods ultimately led to the same result, except for the number of the iterations (Helmert and Bique = 14 iterations, Förstner = 40 iterations)

Levelling	*A priori* levelling standard error τ (mm km$^{-0.5}$)	Variance factor	Re-scaled levelling standard error (mm km$^{-0.5}$)
1	±1.387	3.19138	±2.478
2	±0.423	0.93683	±0.409
3	±1.576	0.95863	±1.543
4	±0.191	1.22798	±0.212

TILT OF LAKE PEIPSI

Lake tilts between water gauge (WG) pairs from monthly mean water level records relative to the Baltic Height System 1977 of Lake Peipsi (Fig. 1) were determined by Raamat (2009) and utilized in the present study. Lake level recordings from 1921 to 2006 from the Estonian as well from the Russian side (Fig. 3) were used for tilt calculation. The shortest observation time series was 10.08 years (Pnevo) and the longest, 83.75 years (Mustvee). The longest gap in the time series was 2 years in Lis′e, Raskopel′ and Zalita. The value of the missing month in the time series was calculated as the average of the same month from the previous and following years to avoid the influence of the seasonal variation in the water level.

Next, 69 combinations of the WG pairs sharing a common observation period were formed and water

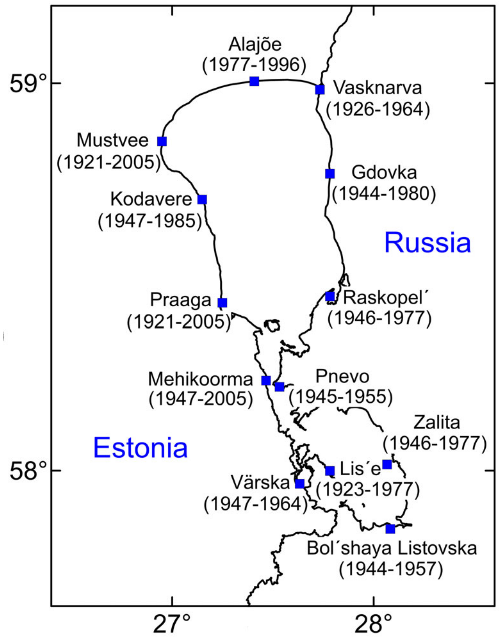

Fig. 3. Water gauges around Lake Peipsi (Fig. 1) and water level observation periods.

level observations were differentiated between them. The shortest and longest common time series between the WG pairs were ~3 years (Gdovka–Alajõe) and 75.42 (Mustvee–Praaga) years, respectively. The slope of the linear trend of the water level differences between the WG pairs reflects the tilt of the lake level, i.e., VV difference between WGs (Mainville & Craymer 2005; Bruxer & Southam 2008). Closing errors of the VV differences in triangles of WGs did not exceed ± 0.56 mm yr^{-1} and the standard error of the VV differences from the closing errors was ± 0.23 mm yr^{-1}.

Strong storms, especially in the autumn–winter period, have significant influence on the water level of Lake Peipsi (Tavast 2009). Therefore, the water level at the WGs can be quite different, which also causes outliers in water level differences between the WG pairs. Outliers in the water level differences were detected and removed in the first iteration visually, trying to raise the value of the determination coefficient R^2 of the trendline. In the second iteration, observations with standard residuals of the regression analysis larger than two $(r_i > 2)$ were removed. The high and low water level cycle of Lake Peipsi is approximately 11 years (Jaani 1973); therefore, the sinusoidal change in water level was also taken into account when removing the outliers.

After removing the outliers, regression analysis of the differentiated water level observations of all WG pairs was repeated. Insignificant slopes $(\alpha = 0.05)$, a total of 19, mostly time series with a length of less than 10 years, were removed from the following calculation. From the significant slopes (a total of 50), 17 slopes with $R^2 > 0.61$ were selected as VV differences for the weighted LSQ adjustment. Similarly to trivial and non-trivial baselines in GPS network processing, correlation exists between WG differences. From 17 selected slopes, five were correlated, i.e., WG differences which use the same observation data form a closed loop. However, these WG differences are only partly correlated, since common observation periods in different WG pairs only partly overlapped. Moreover, values of VVs in LSQ adjustment are not influenced by the correlation between WG pairs. Only the accuracy of VVs is over-estimated.

The LSQ adjustment method was used in pairwise analysis of the WGs. The following observation equation, which accounts for the discrepancy between pairs of the WGs by introducing a residual error, was applied to each pair (Mainville & Craymer 2005):

$$\Delta v_{ij}^{\mathrm{obs}} + r_{ij} = v_i - v_j, \tag{24}$$

where $\Delta v_{ij}^{\mathrm{obs}}$ is the average VV difference of point j relative to point i, from the slope of the linear trend of the water level differences between the WG pairs i and j. The other variables are the output from the LSQ adjustment: v_i and v_j are the VVs of WGs i and j; r_{ij} is the residual error in the observed average VV difference $\Delta v_{ij}^{\mathrm{obs}}$. The $\Delta v_{ij}^{\mathrm{obs}}$ were weighted according to the standard errors of the regression slopes. The VV of the southernmost WG (Bol′shaya Listovska) was fixed to zero to calculate the relative VVs of the WGs. The results of the LSQ adjustment are presented in Table 6.

MODELLING A SURFACE OF VERTICAL CRUSTAL MOVEMENTS FOR ESTONIA

Based on the estimations of S_0^2 and μ (Table 2) and the variance component estimation (Table 5), where data of the first levelling were heavily weighted down, we opted to skip that levelling but otherwise follow Eq. (4) when calculating final velocities for the VCM model. Using only the three latest levellings also made it possible to enclose more BMs in the network, since the second, third and fourth levellings have more common BMs than the CLN of the four levellings. Variance component estimation based on the Bique method using the second, third and fourth levellings gave the levelling standard errors of 0.453, 1.475 and 0.209 mm km$^{-0.5}$, respectively, which only slightly differ from the *a priori* estimates (Table 5).

Levelling is a relative method where only VV differences are estimable. In order to obtain apparent (related to sea level) VVs, relative VVs have to be tied at least with one known apparent velocity. We decided to relate our apparent VVs to the deep-seated BM FR241 in Tallinn with the value of +1.7 mm yr^{-1}. The BM FR241 is placed in limestone and has been used in many previous studies as a stable BM for relating relative VV differences from levellings to the apparent ones. For example, Zhelnin (1958, 1960, 1964, 1966) used the apparent VV value of +2.5 mm yr^{-1} (velocity +1.9 mm yr^{-1} from Bikis (1940) corrected for local subsidence). Vallner & Zhelnin (1975), Vallner (1978) and Vallner et al. (1988) also used same value. Vallner & Zhelnin (1975) and Vallner (1978) used the value of +1.7 mm yr^{-1} for Tallinn, combining repeated levellings with Yakubovski's (1973) VVs for TGs in Kunda and Vormsi for the period 1889–1970. Randjärv (1993) also constrained his map with Yakubovski's (1973) VVs for TGs in Kronstadt, Salacgriva, Liepaja and Baltijsk, and Tallinn from Vallner & Zhelnin (1975). The apparent VV of +1.7 mm yr^{-1} for the Tallinn TG was confirmed also by Davis et al. (1999) based on sea-level observations from 1928 to 1938 from Permanent Service for Mean Sea Level (http://www.psmsl.org/data/). The same apparent VV of +1.7 mm yr^{-1} for FR241 was found also by Kall et al. (2014), where VVs were constrained to the apparent

Table 6. Vertical velocities (VVs) and their standard deviations (both mm yr^{-1}) at the water gauges (WGs) of Lake Peipsi from the weighted least squares (LSQ) adjustment of the lake tilts according to Raamat (2009). Relative VVs are given with respect to the Bolʹshaya Listovska WG whose VV was fixed to zero in the LSQ adjustment. Apparent VVs of the WGs were calculated relative to the Mustvee WG with an apparent VV +0.73 mm yr^{-1}. This is the average apparent VV of the five benchmarks (BMs) of the CLN near the Mustvee WG. Apparent VVs of the BMs were obtained through the kinematic adjustment of the CLN relative to BM FR241 in Tallinn with an apparent VV +1.7 mm yr^{-1}

Water gauge	Latitude (B°)	Longitude (L°)	Relative VV	Standard deviation of the relative VV	Apparent VV
Bolʹshaya Listovska	57.8493	28.0825	0	0	–0.28
Gdovka	58.7663	27.7828	0.97	±0.01	0.69
Lisʹe	57.9994	27.7826	0.23	±0.01	–0.05
Pnevo	58.2161	27.5326	0.52	±0.01	0.24
Raskopelʹ	58.4495	27.7828	0.60	±0.01	0.32
Zalita	58.0160	28.0660	0.21	±0.01	–0.07
Alajõe	59.0050	27.4075	1.17	±0.02	0.89
Kodavere	58.6996	27.1493	0.80	±0.01	0.52
Mehikoorma	58.2328	27.4660	0.36	±0.01	0.08
Mustvee	58.8496	26.9493	1.01	±0.01	0.73
Praaga	58.4329	27.2493	0.58	±0.01	0.30
Värska	57.9661	27.6325	0.22	±0.02	–0.06
Vasknarva	58.9830	27.7327	1.17	±0.01	0.89

velocity of Ristna TG +2.1 mm yr^{-1}. Although we found new VVs for six coastal TGs, the Tallinn TG was not among them, since it has been sinking during most of our calculation period (1960–2010) (see Lutsar 1965; Kall & Torim 2003). There were also large gaps in the time series for that TG in our calculation period.

Outliers among the observations were detected iteratively using the ‘data snooping’ method by Baarda (1968) with the significance level $\alpha = 0.001$. The detected outliers (altogether 41 outliers from 10 iterations) were weighted down using the ‘Danish method’ according to Caspary (2000). Then, the apparent VVs of the BMs relative to deep-seated BM FR241 in Tallinn were found from the LSQ kinematic adjustment with the Bique weights. Post-adjustment statistics $S_0^2 = 0.86$ and $\mu = \pm 0.12$ mm yr^{-1} were obtained.

No direct levelling connections between the WGs of Lake Peipsi and the BMs of the CLN were available for common processing. Therefore, relative VVs of the WGs from the LSQ adjustment of the lake level tilts (fourth column in Table 6) were indirectly connected with the apparent VVs from the kinematic adjustment of the levellings. For that purpose average apparent VV (+0.73 mm yr^{-1}) of the five BMs close to the Mustvee WG (within a radius of 5 km) was assigned to this WG. The VVs of other WGs were shifted relative to it (final column in Table 6).

For the creation of the modelled surface of the VCMs, gridding with the Surfer software (Golden Software Inc.) was used. Several gridding methods were tested. The ‘minimum curvature’ method was chosen based on the residuals of the gridded surface at the observation points, a cross-validation technique and visual appearance. The VCM surface was created for the area 57.45°–59.82°N and 21.67°–28.64°E, with a grid spacing of 2 × 2 km in accordance with the average distance between the closest BM pairs. For removing outliers, BMs deviating more than ±0.3 mm yr^{-1} from the neighbouring ones were visually eliminated from the dataset (altogether 53 BMs from 336), after which the gridded surface was filtered based on the ‘threshold averaging’ method available in Surfer. The obtained VCM model (Fig. 4) was named EST2015LU for referencing purposes.

The main feature of the compiled map (Fig. 4) is the SE–NW directional postglacial land uplift. However, there are also two conspicuous VCM anomalies, one of which is the area surrounding Pärnu (see Fig. 1). The geological setting of Pärnu is characterized by a Proterozoic crystalline basement covered by Silurian limestone and Devonian sandstone. The surface of sedimentary rocks lies at a depth from –10 to –15 m (Tavast & Raukas 1982). These Palaeozoic rocks are covered with Quaternary sediments. The lower layer is loamy till of Late Weichselian age. The loamy till is

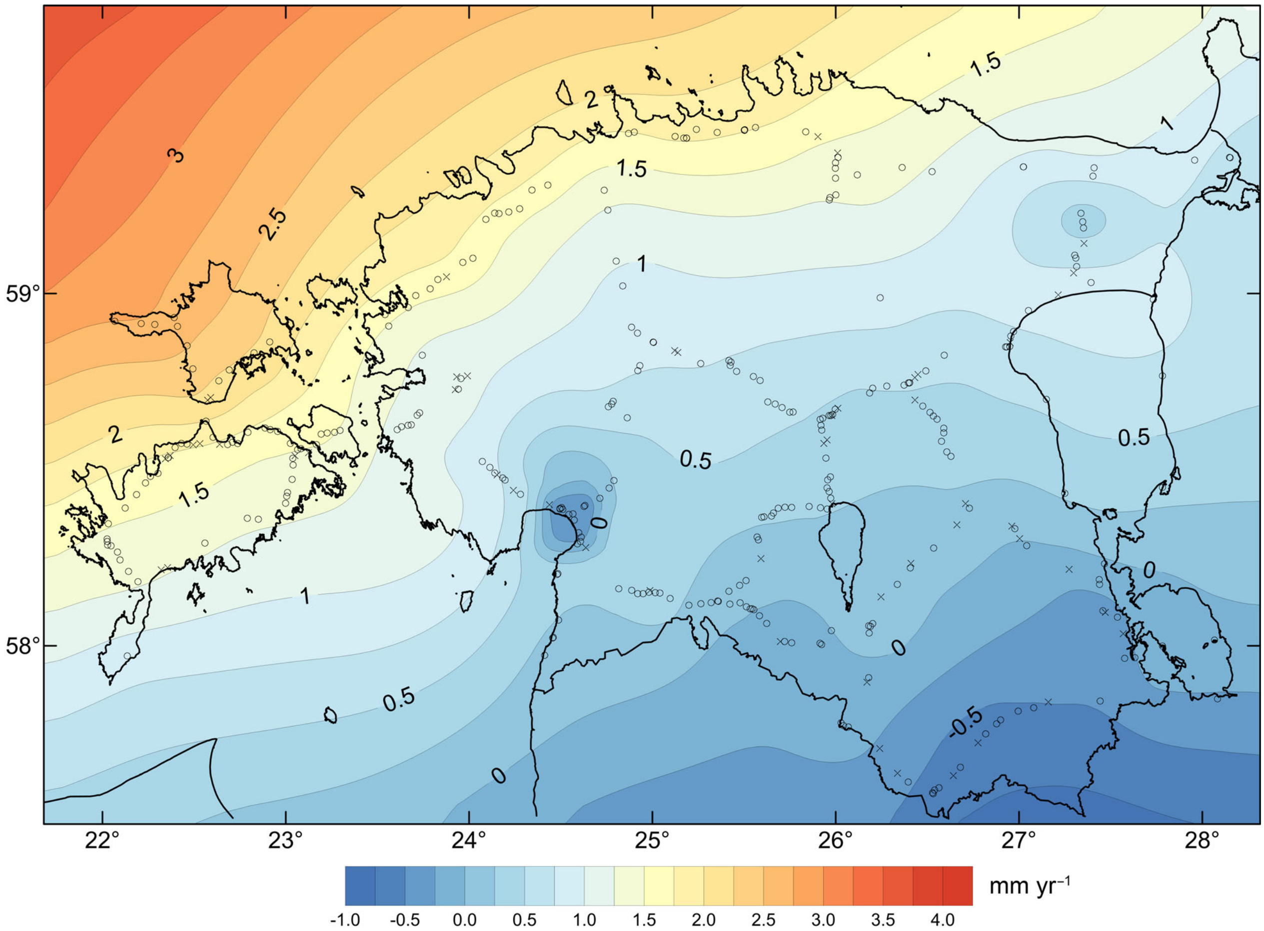

Fig. 4. The model of vertical crustal movements EST2015LU based on 283 apparent vertical velocities (VVs) of the benchmarks (BMs) from the kinematic adjustment of the second, third and fourth levellings of the common levelling network (Fig. 1) relative to BM FR241 in Tallinn with the apparent VV of +1.7 mm yr^{-1}. The apparent VVs of 12 water gauges (WGs) of Lake Peipsi (Table 6) were also used. The contour interval is 0.25 mm yr^{-1}. Average standard errors of the VVs were ±0.12 mm yr^{-1} (BMs) and ±0.01 mm yr^{-1} (WGs). Statistics of model residuals were min –3.07 mm yr^{-1} (BM in Pärnu), max +0.95 mm yr^{-1} (also BM in Pärnu), mean –0.06 mm yr^{-1} and root mean square (RMS) ±0.34 mm yr^{-1}. The model's cross-validation residual RMS error was ±0.30 mm yr^{-1}. The circles indicate the location of BMs and WGs used in the modelling (a total of 295); crosses (x) indicate the location of BMs visually removed before modelling (53).

covered with varved clay or silt with an average thickness of ~10 m. Clay is lacking in only a few areas. Its thickness changes rapidly and is related to irregular topography of the underlying till. The varved clay is covered with a 2–3 m thick layer of Holocene marine and aeolian fine sand and silt which also may be locally absent (Kohv 2011; Talviste et al. 2012; Hang & Kohv 2013). Local subsidence in Pärnu has been related to a decrease in groundwater level. This causes a change in pore pressure, which in turn depends on the thickness of the clay (Listra & Talviste 1988; Mets et al. 2000). The groundwater level in Pärnu has been continuously decreasing since the 1960s. Groundwater pumping achieved its maximum in 1988–1990, causing a lowering of the piezometric level by approximately –10 to –12 m. In 1990–2000 the piezometric level gradually recovered (mean rise +5 m, max 12 m). The piezometric level in Pärnu has been stable since 2001 at an altitude from 0 to 1.5 m with seasonal variation within ±1.5 m (Kohv & Hang 2013). Between 1960 and 1988 the subsidence of the BMs up to –20 mm yr^{-1} was observed (Listra & Talviste 1988). According to recent results, the subsidence of the BMs has stabilized and in one area where the clay is thinner (6 m in average) has even been replaced by uplift (Miller 2013). Our adjustment results show that the sinking of the BMs in Pärnu reach up to –3.5 mm yr^{-1}. The largest residuals of the calculated VVs compared to the smoothed model EST2015LU (Fig. 4) also occur in the Pärnu area. The subsidence of the BMs in Pärnu gradually decreases from the city centre to the suburbs.

A second VCM anomaly in Fig. 4 is in the NE of Estonia. This area is geologically well studied due to the oil shale deposit which has been mined in Estonia for over 90 years. Land surface subsidence occurs above the underground mines when mined-out cavities collapse. The first spontaneous collapse of the underground mine and subsidence of the surface occurred in 1964. Until 2003, 73 collapses have been registered (Pastarus & Sabanov 2005). As a result of these collapses, the surface may subside from 0.7 to 1.5 m, depending on the type of mining (Toomik & Liblik 1998). The residual subsidence of the BMs in that area is most likely related to the mining of the oil shale (Rüdja 2004). According to our study, the residual subsidence of the BMs in relation to the surrounding area is approximately -0.7 mm yr^{-1}.

In Fig. 4, the isolines of VCMs in central Estonia decline more in an east–west direction than in previous land uplift maps of Fennoscandia or Estonia. The declination of the isolines was also noticed in the land uplift model EST2013LU (Kall et al. 2014). However, this direction of the isolines is supported by the VVs of the WGs of Lake Peipsi. In addition, the isolines over Lake Peipsi and SE Estonia on the EST2015LU model (Fig. 4) greatly resemble the isolines of the VCM map by Randjärv (1993), who incorporated levelling data from the Russian side and Latvia into his calculations. Other Estonian VCM maps (e.g., Vallner et al. (1988) and Torim (2004)), where isolines in SE Estonia do not form such shapes, have been compiled based only on Estonian levelling data.

Surfaces of the VCMs from the other levelling combinations were constructed as well. From the comparison of all VCM models it was concluded that data from the first and third levellings influenced the isolines of the VCMs to decline in a SW–NE direction (VCM models based on the VVs of the combinations 1–2–3 and 1–3), whereas data from the second and especially fourth levellings influenced the isolines to decline in a more W–E direction (combinations 1–2–4, 2–3–4 and 2–4). The larger weights of the second and particularly fourth levellings had a larger influence on the isolines of the EST2015LU model, resulting in their declining in a more W–E direction than in earlier maps.

It is obvious that some information about the VVs of the BMs has been lost during the modelling process. In order to highlight that information, the profile of cumulative raw relative VVs of the BMs from SE Estonia (Koidula) to North Estonia (Tallinn) was compiled (Fig. 5). The VVs relative to Koidula were interpolated from the EST2015LU model along this line and added to the profile. The profile of the EST2015LU model follows the profiles of the cumulative VVs between the second and third and the third and fourth levellings most noticeably. Clearly this is related to the fact that the EST2015LU model was based on the second, third and fourth levellings. The fact that the EST2015LU profile follows the cumulative VV profile of the third and fourth levellings better is related to the higher weights of the fourth levelling in the kinematic adjustment.

In addition to the general postglacial tilt, several spikes can be noticed on the profiles. Some spikes are common to all three VV combinations, differing only in the magnitude of the spikes. However, there are also spikes that have happened only in one combination (e.g., 1–2). Spikes in general indicate local anomalies of VCMs. Negative spikes are associated with near-surface

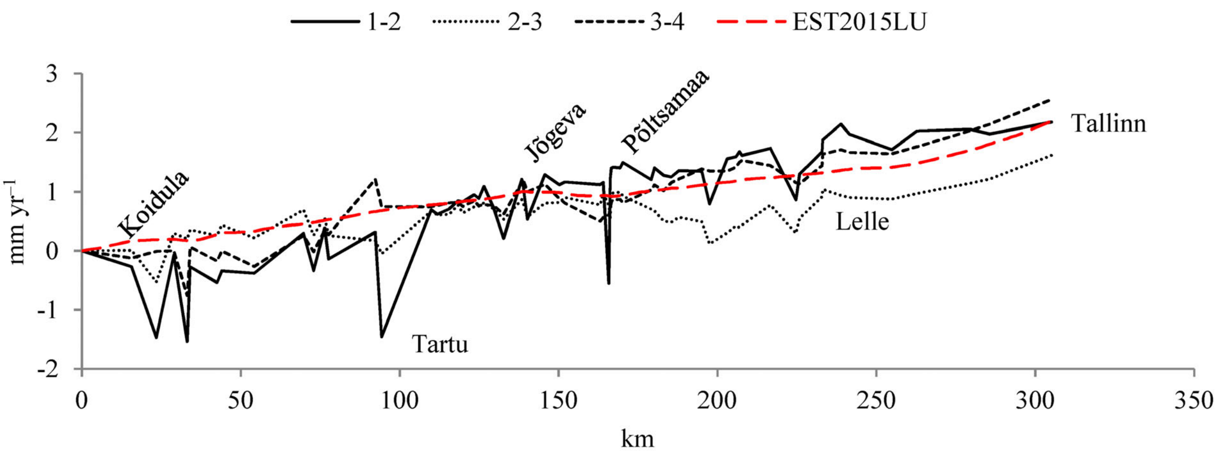

Fig. 5. Profile of the raw (unadjusted) vertical velocities of the benchmarks between the first and second (1–2), second and third (2–3) and third and fourth (3–4) levellings along the Koidula–Tartu–Jõgeva–Põltsamaa–Lelle–Tallinn levelling line (Fig. 1) relative to Koidula. The profile of the EST2015LU model (Fig. 4) along this line relative to Koidula was added for comparison.

processes like groundwater lowering, compression of the sediments, subsidence of buildings, mining, heavy traffic, etc. Positive spikes may result from human activity-related displacements like construction work or levelling errors, but also from near-surface processes like frost heave (Ellenberg 1987; Giménez et al. 2000). Therefore, spikes on the profiles are not related to postglacial rebound or tectonic causes. In Estonia, where most of the country is covered with Quaternary sediments thicker than 5 m, most of the local VCM anomalies are related to the compaction or expansion of sediments.

Steps and changes in the slope on the profiles can be related to geological or tectonic phenomena, such as different compaction of recent sediments, regional tectonic tilt or active tectonic structures (Giménez et al. 2000). Based on this methodology, Vilde (2013) analysed the graphs of the cumulative VVs of four precise levellings of the Estonian levelling network. He concluded that there was a connection between the steps on the cumulative VV profiles and locations of the tectonic faults on the Jõhvi–Tapa, Tallinn–Tapa and Põltsamaa–Lelle levelling lines.

Step-like features in Fig. 5 can be noticed on kilometres 40 and 70 between Koidula and Tartu, as well as between Põltsamaa and Lelle (170–230 km). The Põltsamaa–Lelle line crosses several tectonic faults of the crystalline basement and sedimentary rocks. A relation between VCMs along this line and tectonic faults has been identified in previous studies (Sildvee 1973; Vallner & Zhelnin 1975; Vallner et al. 1988; Kall & Oja 2006; Kall & Jürgenson 2008). Changes in the slope of VVs can also be observed in both abovementioned lines at least in one levelling combination. Most likely it is related to different compaction of the sediments. For example, the Koidula–Tartu levelling line runs intermittently on till and sand of different thicknesses. The relationship between changes in the slope and different compaction of sediments has been shown in a previous study by Zhelnin (1966). For example, he explained the great subsidence of the BMs along the Põltsamaa–Lelle levelling line in 1961–1964 by the drought in 1964, when the groundwater level was extraordinarily low. According to Zhelnin (1966), the drought had no influence for the first 20 km of the levelling line because sedimentary rocks only lie at a depth of 1.3 m. The subsidence of the BMs appeared for further sections of the line because the thickness of the Quaternary sediments also increased.

Several attempts have been made to relate VCMs from the repeated levellings to the block structure of the crystalline basement and sedimentary cover of Estonia. However, no firm connection between the VCMs and block structure of the crystalline basement has been found (Sildvee & Miidel 1978, 1980). Five velocity planes of VCMs for Estonia (southeastern, middle, northeastern, northwestern and western Saaremaa) were calculated by Vallner et al. (1988). These planes move parallel or under a small angle relative to each other and coincide with the regional gravity structures. Vallner et al. (1988) concluded that VCMs follow block movements in Estonia. Nevertheless, based on the comparison between the VCM map and the block structure of the crystalline basement (Pobul & Sildvee 1975), Vallner et al. (1988) came to the same conclusion as the former authors that there is no clear correlation between them. They found no accordance between the VCMs and structure of the sedimentary cover either.

COMPARISON WITH VERTICAL VELOCITIES FROM THE CONTINUOUSLY OPERATING GNSS REFERENCE STATIONS AND COMBINATION OF TIDE GAUGE OBSERVATIONS AND SATELLITE ALTIMETRY

The EST2015LU model was compared with the VVs of the seven continuously operating Global Navigation Satellite System (GNSS) reference stations (CORS) in Estonia as reported by Oja et al. (2014). The absolute VVs of the CORS (v_{ABS}) were calculated in an ITRF2005 reference frame and based on a 6.5-year time period (November 2007 to May 2014). From the EST2015LU model, apparent VVs (v_{APP}) related to sea level were interpolated to the CORS. In order to compare absolute VVs with apparent ones, the following equation was employed:

$$v_{\mathrm{ABS}} = (a + v_{\mathrm{APP}}) \cdot b, \tag{25}$$

where a is secular sea level rise (eustatic rise) and b is a dimensionless variable describing the change in the geoid. In the present study b was fixed to the value 1.06 (Vestøl 2006), which corresponds to about 6% (~0.6 mm yr^{-1}) of the Fennoscandian land uplift maximum (~10 mm yr^{-1}) at the Gulf of Bothnia. A similar value for the geoid change was also found by Tamisiea et al. (2002) and Ekman & Mäkinen (1996), 0.5 mm yr^{-1} and 0.6 mm yr^{-1}), respectively.

The value of a was found from the VV difference $v_{\mathrm{ABS}} - v_{\mathrm{APP}}$ of the six CORS from a LSQ solution that minimized the velocity residuals. Using this method, the value of a was 2.11 ± 0.14 mm yr^{-1}. A similar value was also obtained by Oja et al. (2014), using the same methodology of employing v_{APP} values interpolated to CORS from different land uplift models. Residual VVs of the CORS obtained from Eq. (25) are presented in Fig. 6.

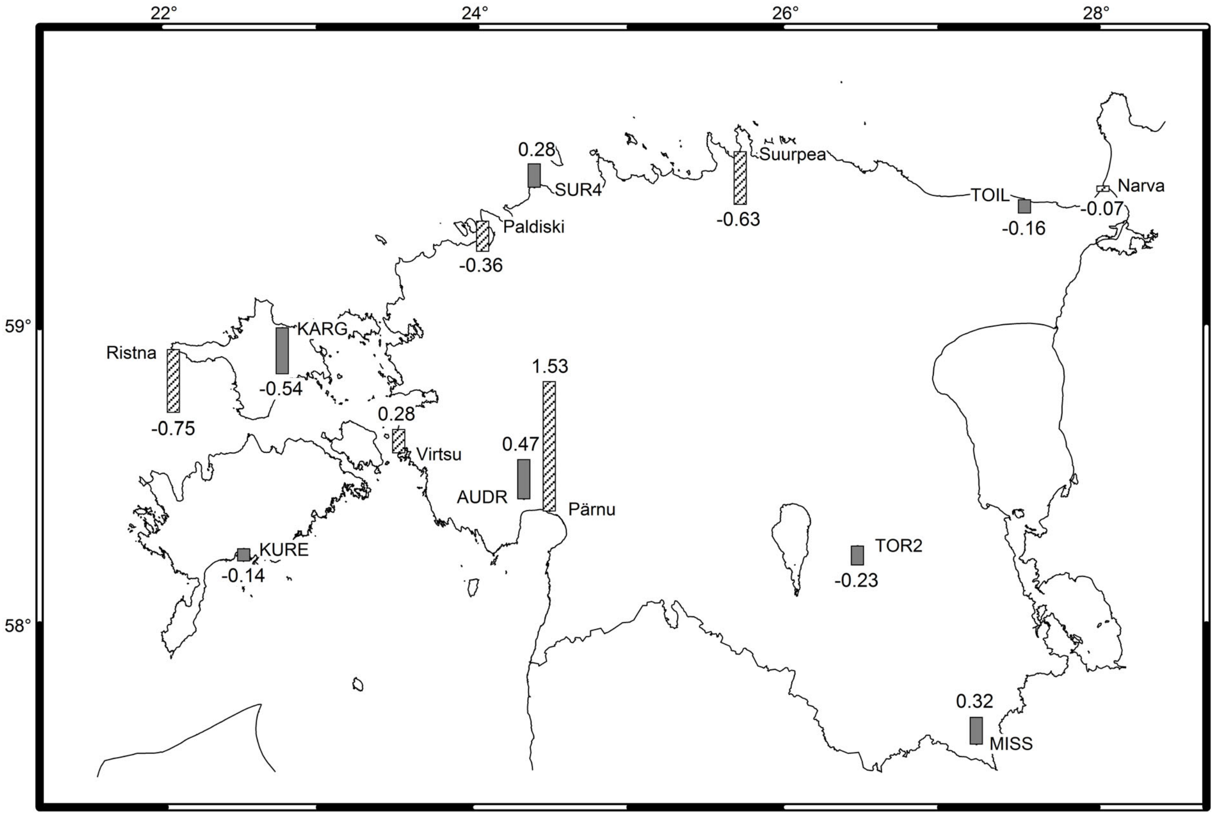

Fig. 6. Residual vertical velocities (VVs, in mm yr^{-1}) (observed–interpolated) from the comparison of the GNSS-based (grey bars) and tide gauge and satellite altimetry (TG & SA)-based absolute VVs (patterned bars) with the apparent VVs interpolated from the EST2015LU model (Fig. 4). The root mean square values of the residual VVs were ±0.34 mm yr^{-1} (GNSS) and ±0.76 mm yr^{-1} (TG & SA; ±0.48 mm yr^{-1} without Pärnu).

The comparison of the EST2015LU model with GNSS data gave the RMS of the residual VVs ±0.34 mm yr^{-1}. The largest residuals occur in the Pärnu area and on the Island of Hiiumaa (AUDR and KARG sites; Fig. 6). The larger difference in the Pärnu area could be explained by the fact that levelling-based VVs cover the period when this area was subsiding, while GNSS observations were performed when subsidence had stopped. The difference on the Island of Hiiumaa may result from errors in levelling-based VVs, since Hiiumaa is connected to the CLN only from the one side with the hydrostatic or hydrodynamic levelling between Saaremaa and Hiiumaa (Fig. 1). The differences also result in part from the choice of the reference apparent VV for the kinematic adjustment of the CLN.

Because of systematic differences, historical VVs for Estonian TGs (Yakubovski 1973; Pobedonostsev 1975; Jevrejeva et al. 2002) cannot be used for the evaluation of land uplift models (Kall et al. 2014). For the present study new VVs for Estonian coastal TGs were calculated using a different methodology. The sea level observation period was shorter (1960–2010) than in previous studies and TG data were corrected using satellite altimetry (SA) data (1992–2013). The methodology used was based on the studies by Kuo et al. (2004, 2008). Absolute VVs (v_{ABS}) were found for six TGs. Apparent VVs (v_{APP}) were interpolated to TGs from the EST2015LU model in Fig. 4. Absolute VVs were compared with apparent VVs as well as with GNSS velocities. The value for sea level rise $a = 1.49 \pm 0.34$ mm yr^{-1} was obtained. Residual VVs of the TGs based on Eq. (25) are presented in Fig. 6.

The comparison of VVs from the EST2015LU model with the TG & SA data yielded the RMS of the residual VVs ±0.76 mm yr^{-1}. This is slightly better than that found by Kall et al. (2014). Like with GNSS data, the largest residuals occur in Pärnu and on the Island of Hiiumaa but also in North Estonia (Pärnu, Ristna and Suurpea sites; Fig. 6). The reasons for these biases are most likely the same as previously discussed. The fit with the VVs of TGs is much better without Pärnu. Residual VV RMS without Pärnu was ±0.48 mm yr^{-1}.

This discrepancy is within the limits of uncertainty, considering the accuracy of the VV determination from levellings and TG & SA.

At the same time, the comparison between the CORS and TG absolute VVs showed that differences were even larger than with the EST2015LU model (RMS = ± 0.8 mm yr^{-1}). The VVs of GNSS were systematically larger than those of the TGs. In our opinion, the main reason for the difference is related to the reference frames used. For TG & SA VVs the frame is defined by the orbit of the TOPEX/POSEIDON mission. The CORS VVs are aligned to ITRF2005. Santamaría-Gómez et al. (2014) have pointed out that the VV differences between the estimates from the Kuo et al. (2004) method and BIFROST GPS VVs in the Baltic Sea remain between -2.0 and 0.5 mm yr^{-1}: on the latitude of Estonia the differences are between -0.8 and 0.5 mm yr^{-1}. However, the errors of our method and GNSS may also be similar to the order of ± 0.8 mm yr^{-1} as we are using an indirect method to estimate VVs with the geophysical assumption that TG and SA observed the identical geocentric sea level rise. Another assumption is that only linear vertical motion exists, as the time span of CORS is much smaller than that for TG.

SUMMARY AND CONCLUSIONS

The aim of this study was to evaluate VCMs in Estonia over time, based on a CLN created from a dataset of four precise levellings, and from water level observations of Lake Peipsi. The difference between *a posteriori* variances of unit weight between three or four and two levelling combinations indicates that the VVs of the BMs between the levelling periods have been uneven, observations contained errors, or weight matrices of the observations did not fit together in the kinematic adjustment of more than two levelling combinations. Estimates of the mean standard error of VV differences were mostly dependent on whether the fourth levelling was involved in the adjustment. The mean standard error of the VV differences was also influenced by the time period between levellings. The larger the time period, the smaller the standard error was.

Neither variances of unit weight nor mean standard errors of the VV differences differed significantly between the models parameterizing VVs with and without heights, regardless of levelling combinations used. In most levelling combinations, except 1–2–3 and 1–2, there were also no significant differences between the VVs from the 'heights included' and 'heights eliminated' models. Therefore, levelling observations did not contain systematic errors which would be removed from the dataset by differencing the observations.

A significant difference between VVs $\mathbf{v}_{1-2}$, $\mathbf{v}_{2-3}$, $\mathbf{v}_{3-4}$ was found. This means that VVs of the BMs changed over time. The change in VV can either be a real change or fortuitous, caused by levelling errors or correlation between the levellings. Since a strong significant correlation (0.691, $p < 0.05$) between the second and third levellings was detected, it remained unresolved whether the VV change was real or dependent on the between-epoch correlation of the second and third levellings. Between-epoch correlation influences deformation analyses, independently of whether the correlation is fortuitous or genuine (Mäkinen & Saaranen 1998).

The detected VV change over time could also be explained by levelling errors, if it is assumed that the levelling error was 3.2 times larger than indicated by loops' misclosures. The increase could be divided uniformly to the four levellings but iterated variance component estimation put most of the error on the first levelling.

Apparent VVs from the kinematic adjustment of the combination of the second, third and fourth levellings, as the mathematically best fitting observations with rescaled weights along with the apparent VVs of the WGs of Lake Peipsi (Table 6) were used to draw the VCM model EST2015LU (Fig. 4). Compared to earlier VCM maps for Estonia and Fennoscandia, the isolines of EST2015LU over Lake Peipsi and SE Estonia resemble most the VCM map by Randjärv (1993). The EST2015LU model also highlighted two VCM anomalies. The first, in Pärnu, is related to the compaction of the sediments due to groundwater withdrawal. The second one is located in NE Estonia and is probably related to oil shale mining in that area.

The comparison of absolute and apparent VVs of seven CORS, based on Eq. (25), yielded an average residual VV of ± 0.34 mm yr^{-1}. Taking the average standard deviation of the VVs and the errors of EST2015LU into account, the fit between the VVs from the EST2015LU model and CORS can be considered good.

Absolute VVs for six TGs based on sea level observations from 1960 to 2010 were calculated and corrected by using SA data (1992–2013). From the comparison of the TG & SA VVs with the ones interpolated from the EST2015LU model, an average residual VV of ± 0.76 mm yr^{-1} was found. Larger residuals on the Island of Hiiumaa (with both CORS and TG & SA VVs) suggest that height connections between the Islands of Saaremaa and Hiiumaa should be re-examined. At the same time, the differences between the CORS and TG VVs were even greater (RMS = ± 0.8 mm yr^{-1}), whereas VVs from the CORS were systematically larger. The main reason for the CORS and TG & SA VV difference is related to the reference frames used.

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REFERENCES

Amiri-Simkooei, A. 2007. *Least-squares Variance Component Estimation: Theory and GPS Applications* [PhD thesis]. Delft University of Technology, Publication on Geodesy, 64, Netherlands Geodetic Commission, Delft, 208 pp.

Baarda, W. 1968. *A Testing Procedure for Use in Geodetic Networks*. Netherlands Geodetic Commission, Delft, 97 pp.

Bähr, H., Altamimi, Z. & Heck, B. 2007. *Variance Component Estimation for Combination of Terrestrial Reference Frames*. Universität Karlsruhe (TH), Karlsruhe, 69 pp.

Bikis, J. 1940. *Zemes garozas kustības ietekme precīzās līmetņošanas darbos Baltijas jūras piekrastē* [*Influence of the Earth's Crust Movements on the Levelling Work Precision on the Coast of the Baltic Sea*]. Riga, Latvijas Universitātes raksti, ser. II, No. 6, 201 pp. [in Latvian].

Brown, L. D. & Oliver, J. E. 1976. Vertical crustal movements from leveling data and their relation to geologic structure in the eastern United States. *Reviews of Geophysics*, **14**, 13–35.

Bruxer, J. & Southam, C. 2008. *Review of Apparent Vertical Movement Rates in the Great Lakes Region*. Burlington, Report prepared for the International Upper Great Lakes Study, HY-1D, 88 pp.

Carrera, G. H. & Vaníček, P. 1986. Review of techniques for determining vertical ground movements from levelling data. In *Proceedings of Third International Symposium on Land Subsidence*, pp. 195–202. IAHS Publication, Venice.

Carrera, G. H., Vaníček, P. & Craymer, M. R. 1991. *The Compilation of a Map of Recent Vertical Crustal Movements in Canada*. Department of Surveying Engineering, University of New Brunswick, Fredericton, N.B., Technical Report, 153, 107 pp.

Caspary, W. 2000. *Concepts of Network and Deformation Analysis* (Rüeger, J. M., ed.). University of New South Wales, Sydney, Australia, 187 pp.

Cross, P. A., Hannah, J., Hradilek, L., Kelm, R., Mäkinen, J., Merry, C. L., Sjöberg, L. E., Steeves, R. R., Vaníček, P., & Zilkoski, D. B. 1987. Four-dimensional geodetic positioning. *Manuscripta Geodaetica*, **12**, 147–222.

Davis, J. L., Mitrovica, J. X., Scherneck, H.-G. & Fan, H. 1999. Investigations of Fennoscandian glacial isostatic adjustment using modern sea level records. *Journal of Geophysical Research: Solid Earth*, **104**(B2), 2733–2747.

Ekman, M. & Mäkinen, J. 1996. Recent postglacial rebound, gravity change and mantle flow in Fennoscandia. *Geophysical Journal International*, **126**, 229–234.

Ellenberg, J. 1987. The geological treatment of relevellings. *Journal of Geodynamics*, **8**, 235–243.

Förstner, W. 1979. Ein Verfahren zur Schätzung von Varianz- und Kovarianzkomponenten. *Allgemeine Vermessung Nachrichten*, **86**(11–12), 446–453.

Ghilani, C. D. & Wolf, P. R. 2006. *Adjustment Computations: Spatial Data Analysis*. John Wiley & Sons, Hoboken, N.J., 611 pp.

Giménez, J., Suriñach, E. & Goula, X. 2000. Quantification of vertical movements in the eastern Betics (Spain) by comparing levelling data. *Tectonophysics*, **317**, 237–258.

Hang, T. & Kohv, M. 2013. Glacial varves at Pärnu, southwestern Estonia: a local varve chronology and proglacial sedimentary environment. *GFF*, **135**, 273–281.

Hein, G. W. 1986. *A Model Comparison in Vertical Crustal Motion Estimation Using Leveling Data*. National Oceanic and Atmospheric Administration, National Ocean Service, Charting and Geodetic Services, Rockville, Md., 46 pp.

Helmert, F. R. 1872. *Die Ausgleichungsrechnung nach der Methode der kleinsten Quadrate: mit Anwendungen auf die Geodäsie und die Theorie der Messinstrumente*. B. G. Teubner, 370 pp.

Holdahl, S. H. 1978. Models for extracting vertical crustal movements from leveling data. In *Proceedings of the GEOP Conference "Applications of Geodesy to Geodynamics". October 2–5, 1978*, pp. 183–190. Ohio State University, Columbus, Ohio.

Jaani, A. 1973. Veerohkus muutub tsükliliselt [Cyclic changes in the abundance of water]. *Eesti Loodus*, 24, 758–764 [in Estonian].

Jevrejeva, S., Rudja, A. & Makinen, J. 2002. Postglacial rebound in Fennoscandia: new results from Estonian tide gauges. In *Gravity Geoid and Geodynamics 2000* (Sideris, M. G., ed.), pp. 193–198. Springer-Verlag Berlin, Berlin.

Kääriäinen, E. 1953. *On the Recent Uplift of the Earth's Crust in Finland*. Finnish Geodetic Institute, Helsinki, Suomen geodeettisen laitoksen julkaisuja, 42, 106 pp.

Kääriäinen, E. 1963. Land uplift in Finland computed by the aid of precise levellings. *Fennia*, **89**, 15–18.

Kakkuri, J. & Vermeer, M. 1985. *The Study of Land Uplift Using the Third Precise Levelling of Finland*. Finnish Geodetic Institute, Helsinki, Reports of the Finnish Geodetic Institute, 85:1, 11 pp.

Kall, T. & Jürgenson, H. 2008. Postglacial land uplift in Estonia based on geodetic measurements on Põltsamaa-Lelle levelling line. In *The 7th International Conference "Environmental Engineering", May 22–23, 2008, Selected Papers* (Cygas, D. & Froehner, K. D., eds), pp. 1325–1333. Vilnius Gediminas Technical University Press, Technika, Vilnius.

Kall, T. & Oja, T. 2006. Geodetic and geophysical repeated measurements in geodynamic monitoring networks of Estonia. In *Geodetic Deformation Monitoring: From Geophysical to Engineering Roles, International Association of Geodesy Symposia* (Sanso, F. & Gil, A. J., eds), pp. 222–230. Springer-Verlag Berlin, Jaen, Spain.

Kall, T. & Torim, A. 2003. Vertical movements on the territory of Tallinn. *Journal of Geodynamics*, **35**, 511–519.

Kall, T., Oja, T. & Tänavsuu, K. 2014. Postglacial land uplift in Estonia based on four precise levelings. *Tectonophysics*, **610**, 25–38.

Kalm, V. 2007. The urban geology of Tartu, Estonia. In *Applied Quaternary Research in the Central Part of Glaciated Terrain. Proceedings of the INQUA Peribaltic*

Group Field Symposium 2006, Oulanka Biological Research Station, Finland, September 11.–15. (Johansson, P. & Sarala, P., eds), *Geological Survey of Finland, Special Paper*, 46, 141–145.

Koch, K. 1978. Schätzung von varianzkomponenten. *Allgemeine Vermessungs Nachrichten*, **85**, 264–269.

Koch, K.-R. 2010. *Parameter Estimation and Hypothesis Testing in Linear Models*. Springer, Berlin, Heidelberg, 334 pp.

Kohv, M. 2011. *Landslides in Clayey Soils of Western Estonia* [Doctoral dissertation]. University of Tartu, Tartu, 116 pp.

Kohv, M. & Hang, T. 2013. Pore pressure fluctuations within quasi-stable slopes in south-western Estonia and their influence on slope stability. In *Landslide Science and Practice* (Margottini, C., Canuti, P. & Sassa, K., eds), pp. 79–85. Springer, Berlin, Heidelberg.

Krarup, T., Juhl, J. & Kubik, K. 1980. Gotterdammerung over least squares adjustment. In *International Archives of Photogrammetry* (Ackermann, F., Bauer, H., Konecny, G. & Kupfer, G., eds), pp. 369–378. Committee of the XIV International Congress for Photogrammetry and Remote Sensing, Hamburg.

Kuo, C. Y., Shum, C. K., Braun, A. & Mitrovica, J. X. 2004. Vertical crustal motion determined by satellite altimetry and tide gauge data in Fennoscandia. *Geophysical Research Letters*, **31**, L01608 1–4.

Kuo, C.-Y., Shum, C. K., Braun, A., Cheng, K.-C. & Yi, Y. 2008. Vertical motion determined using satellite altimetry and tide gauges. *Terrestrial, Atmospheric and Oceanic Sciences*, **19**, 21–35.

Listra, E. & Talviste, P. 1988. *Geotehniline kontroll nõrkade savipinnaste levialal. I köide. Pinnaseomadused ja vajumisvaatluste analüüs Pärnus* [*Geotechnical Control in the Areas of Weak Surfaces of Clay and Till. Vol. 1. Subsidence Observations in Pärnu*]. National Institute of Building Research, Tallinn, Report from the database of Estonian geological engineering, Thematical research, 24781, 219 pp. [in Estonian].

Lucas, J. R. 1985. *A Variance Component Estimation Method for Sparse Matrix Applications*. Rockville, MD, NOAA Technical Report; NOS 111 NGS 33, 12 pp.

Lutsar, R. 1965. Smeshchenie reperov nivelirnoj seti goroda Tallina [Displacements of the Bench Marks of the Levelling System Network of the City of Tallinn]. In *Sovremennye dvizheniya zemnoj kory, N 2* [*Recent Crustal Movements, No. 2*] (Parfjonova-Riives, L. J., ed.), pp. 288–293. Academy of Sciences of the Estonian SSR, Tartu [in Russian, with English summary].

Lutsar, R., Saapar, L. & Arbeiter, R. 1973. Izuchenie dvizhenij zemnoj poverkhnosti v gorodakh Éstonskoj SSR [Investigation of the crustal movements within the precinct of towns of the Estonian SSR]. In *Sovremennye dvizheniya zemnoj kory, N 5* [*Recent Crustal Movements, No. 5*], pp. 139–143. Estonian Academy of Sciences, Tartu [in Russian, with English summary].

Mainville, A. & Craymer, M. R. 2005. Present-day tilting of the Great Lakes region based on water level gauges. *Geological Society of America Bulletin*, **117**, 1070–1080.

Mäkinen, J. 2002. A bound for the Euclidean norm of the difference between the best linear unbiased estimator and a linear unbiased estimator. *Journal of Geodesy*, **76**, 317–322.

Mäkinen, J. & Saaranen, V. 1998. Determination of post-glacial land uplift from the three precise levellings in Finland. *Journal of Geodesy*, **72**, 516–529.

Mets, M., Torn, H. & Saapar, L. 2000. Perspectives and problems of geotechnical monitoring in Estonia. In *Proceedings of the Ninth Baltic Geotechnical Conference “Baltic Geotechnics IX 2000”* (Mets, M., ed.), pp. 226–233. Estonian Geotechnical Society, Tallinn.

Miller, M. 2013. *Maapinna vertikaalliikumise iseloom Pärnu linnas perioodil 1987–2010 põhjavee survetaseme muutuste taustal* [*Ground Surface Vertical Movements on the Territory of Pärnu in 1987–2010*]. Master’s thesis. Tartu Ülikool, Tartu, 41 pp. [in Estonian, with English summary].

Oja, T., Kollo, K. & Pihlak, P. 2014. GIAst ja maapinna liikumistest Eestis GNSS täppismõõtmiste valguses [About GIA and Earth’s surface movements in Estonia in the light of precise GNSS measurements]. *Geodeet*, **44**, 55–65 [in Estonian].

Pastarus, J.-R. & Sabanov, S. 2005. A method for securing working mining block stability in Estonian oil shale mines. *Proceedings of the Estonian Academy of Sciences. Engineering*, **11**, 59–68.

Pobedonostsev, S. 1975. Analiz vertikal′nykh dvizhenij poberezh′ya Pribaltiki [Analysis of vertical crustal movements on the coast of the Baltic Sea]. In *Sovremennye dvizheniya territorii Pribaltiki* [*Recent Crustal Movements on the Territory of Baltic States*] (Riives, L., ed.), pp. 28–36. Academy of Sciences of the Estonian SSR, Tartu [in Russian, with English summary].

Pobul, E. & Sildvee, H. 1975. O blokovom stroenii kristallicheskogo fundamenta Éstonii [About the block structure of crystalline basement in Estonia]. In *Sovremennye dvizheniya territorii Pribaltiki* [*Recent Crustal Movements on the Territory of Baltic States*] (Riives, L., ed.), pp. 64–73. Academy of Sciences of the Estonian SSR, Tartu [in Russian, with English summary].

Raamat, R. 2009. *Maakoore vertikaalliikumised Peipsi järve veevaatlusandmete põhjal* [*Land Uplift on Lake Peipsi Using Water Level Observation Data*]. Master’s thesis in Geodesy, Eesti Maaülikool, Tartu, 85 pp. [in Estonian, with English summary].

Randjärv, J. 1968. Kharakter i skorosti sovremennykh dvizhenij zemnoj kory v Pribaltike [The character and velocities of recent crustal movements in Baltic Regions]. In *Sovremennye dvizheniya zemnoj kory, N 3* [*Recent Crustal Movements of the Earth, No. 3*] (Gerasimov, I., ed.], pp. 200–210. USSR Academy of Sciences, Moscow [in Russian, with English summary].

Randjärv, J. 1993. *Vertical Movements of the Earth’s Crust in the Baltic Region*. Finnish Geodetic Institute, Helsinki, Reports of the Finnish Geodetic Institute, 93:2, 33 pp.

Rao, C. 1971. Estimation of variance and covariance components – MINQUE theory. *Journal of Multivariate Analysis*, **1**, 257–275.

Rüdja, A. 2004. *Geodetic Datums, Reference Systems and Geodetic Networks in Estonia*. Doctoral dissertation. University of Helsinki, Helsinki, 331 pp.

Santamaría-Gómez, A., Gravelle, M. & Wöppelmann, G. 2014. Long-term vertical land motion from double-differenced tide gauge and satellite altimetry data. *Journal of Geodesy*, **88**, 207–222.

Sildvee, H. 1973. O svyazi mezhdu razlomov i sovremennymi differentsirovannymi dvizheniyami zemnoj kory na territorii Éstonii [About the connection between the tectonic faults and recent differential vertical movements of the Earth's crust in Estonia]. In *Sovremennye dvizheniya zemnoj kory na geodinamicheskikh poligonakh* [*Recent Crustal Movements on the Geodynamic Polygons*] (Feofilova, V. G., ed.), pp. 173–177. K. I. Satapaev Institute of Geology of the Academy of Sciences of the Kazakhstan SSR, Alma-Ata [in Russian].

Sildvee, H. & Miidel, A. 1978. Nekotorye rezul′taty interpretatsii sovremennykh dvizhenij zemnoj kory v Éstonii [Some results of the interpretation of recent crustal movements in Estonia]. In *Sovremennye dvizheniya zemnoj kory (issledovaniya na geodinamicheskikh poligonakh)* [*Recent Vertical Crustal Movements (Studies on Geodynamic Polygons)*] (Fotiadi, E., ed.), pp. 138–143. Nauka, Novosibirsk [in Russian].

Sildvee, H. & Miidel, A. 1980. Opyt primeneniya geologo-geofizicheskikh metodov dlya interpretatsii sovremennykh dvizhenij zemnoj poverkhnosti na territorii Éstonii [An experience of the application of geological-geophysical methods to interpretation of recent crustal movements on the Estonian territory]. In *Sovremennye dvizheniya zemnoj kory. Teoriya, metody, prognoz* [*Recent Crustal Movements. Theory, Methods, Prognosis*] (Bulanže, Y., Linnenberg, D. & Podstrigach, Y., eds), pp. 110–114. Nauka, Moscow [in Russian].

Talviste, P., Hang, T. & Kohv, M. 2012. Glacial varves at the distal slope of Pandivere–Neva ice-recessional formations in western Estonia. *Bulletin of the Geological Society of Finland*, **84**(Special Issue 1), 7–19.

Tamisiea, M. E., Mitrovica, J. X., Tromp, J. & Milne, G. A. 2002. Present-day secular variations in the low-degree harmonics of the geopotential: sensitivity analysis on spherically symmetric earth models. *Journal of Geophysical Research B: Solid Earth*, **107**, ETG 18-1–18-10.

Tavast, E. 2009. Changing factors of the coasts of Lake Peipsi, North-Eastern Europe. *Quaternary International*, **207**, 130–136.

Tavast, E. & Raukas, A. 1982. *Rel′ef korennykh porod Éstonii* [*The Bedrock Relief of Estonia*]. Valgus, Tallinn, 192 pp. [in Russian].

Toomik, A. & Liblik, V. 1998. Oil shale mining and processing impact on landscapes in north-east Estonia. *Landscape and Urban Planning*, **41**, 285–292.

Torim, A. 2004. Maakoore tõusust ja rannajoone muutusest Eestis [About the land uplift and variation of the coastline in Estonia]. *Geodeet*, **28**, 57–62 [in Estonian].

Vallner, L. 1978. *Izuchenie blokovogo kharaktera sovremennykh dvizhenij territorii Éstonii geodezicheskim metodom* [*The Study of the Block Structure of Recent Crustal Movements of the Estonian Territory by the Geodetic Method*]. PhD dissertation, Shmidt Institute of Physics of the Earth, Moscow [in Russian].

Vallner, L. & Zhelnin, G. 1975. Novaya karta izobaz territorii ÉSSR [New map of isobases on the territory of the Estonian S.S.R.]. In *Sovremennye dvizheniya territorii Pribaltiki* [*Recent Crustal Movements on the Territory of Baltic States*] (Riives, L., ed.), pp. 48–57. Academy of Sciences of Estonian S.S.R., Tartu [in Russian, with English summary].

Vallner, L., Sildvee, H. & Torim, A. 1988. Recent crustal movements in Estonia. *Journal of Geodynamics*, **9**, 215–223.

Vaníček, P. 1976. Pattern of recent vertical crustal movements in Maritime Canada. *Canadian Journal of Earth Sciences*, **13**, 661–667.

Vaníček, P., Castle, R. O. & Balazs, E. I. 1980. Geodetic leveling and its applications. *Reviews of Geophysics*, **18**, 505–524.

Vestøl, O. 2006. Determination of postglacial land uplift in Fennoscandia from leveling, tide-gauges and continuous GPS stations using least squares collocation. *Journal of Geodesy*, **80**, 248–258.

Vilde, S. 2013. *Eesti kõrgusvõrgu reeperite stabiilsuse hindamine graafilisel meetodil* [*Evaluation of Stability of Benchmarks in the Estonian Vertical Network*]. Master's thesis in Geodesy, Eesti Maaülikool, Tartu, 92 pp. [in Estonian, with English summary].

Welsch, W. 1978. A posteriori Varianzschätzung nach Helmert. *Allgemeine Vermessungs Nachrichten*, **85**, 55–63.

Welsch, W. 1984. Grundlagen, Gebrauchsformeln und Anwendungsbeispiele der Schätzung von Varianz- und Kovarianzkomponenten. *Vermessung, Photogrammetrie und Kulturtechnik*, **82**, 296–301.

Witting, R. 1922. Le soulèvement récent de la Fennoscandie. *Geografiska Annaler*, **4**, 458–487.

Yakubovski, O. 1973. Utochnennaya karta vertikal′nykh dvizhenij zemnoj kory poberezh′ya Baltijskogo morya [Renewed map of vertical movements of the Earth's crust on the coasts of the Baltic Sea]. In *Sovremennye dvizheniya zemnoj kory, N 5* [*Recent Crustal Movements, No. 5*], pp. 72–78. Academy of Sciences of the Estonian SSR, Tallinn [in Russian, with English summary].

Zhelnin, G. 1958. Maakoore kerkimine Eestis [Uplift of the Earth's crust in Estonia]. *Eesti Loodus*, 5, 269–274 [in Estonian].

Zhelnin, G. 1960. Izuchenie vertikal′nykh dvizhenij zemnoj kory v ÉSSR metodom povtornykh nivelirovok [Investigations of vertical displacements of the Earth's crust on the territory of the Estonian S.S.R. by the Method of Repeated Levellings]. In *Materialy soveshchaniya po voprosam neotektonicheskikh dvizhenij v Pribaltike* [*Proceedings of the Conference About the Neotectonic Movements in the Baltic Region*] (Orviku, K., ed.), pp. 76–87. Academy of Sciences of the Estonian S.S.R., Tartu [in Russian, with English summary].

Zhelnin, G. 1964. Tochnost′ i vozmozhnosti metoda povtornogo nivelirovaniya [The accuracy of and prospects for the method of repeated levellings]. In *Sovremennye i novejshie dvizheniya zemnoj kory v Pribaltike* [*Recent and Latest Crustal Movements in the Baltic Region*] (Gudelis, V., ed.), pp. 17–24. Department of Geography, Academy of Sciences of the Lithuanian SSR, Vilnius [in Russian, with English summary].

Zhelnin, G. 1966. On the recent movements of the Earth's surface in the Estonian SSR. *Annales Academiae Scientiarum Fennicae. Series A III Geologica-Geographica*, **90**, 489–493.

Maapinna vertikaalliikumised Eestis, tuginedes täppisnivelleerimistele ja Peipsi järve veetaseme vaatlustele

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Eesti kõrgusvõrk on praeguseks ajaks kõrgtäpsete nivelleerimistega mõõdetud juba neli korda (nivelleerimiskampaaniate keskmised epohhid 1936,7, 1961,2, 1982,1 ja 2006,9 aastat). Käesoleva uurimuse eesmärk on hinnata Eesti kõrgusvõrgu reeperite vertikaalliikumise kiirusi ja nende muutust ajas nelja nivelleerimise (periood 1933–2011) andmete põhjal. Vertikaalliikumise kiiruste arvutamiseks kasutati vähimruutude meetodil parameetrilist kinemaatilist tasandamist. Rakendati kaht matemaatilist mudelit, kus esimeses olid parameetriteks reeperite kõrgused ja kiirused ning teises ainult kiirused. Viimased leiti nivelleerimiste üheteistkümnest kombinatsioonist. Erinevate matemaatiliste mudelite kiirused erinesid ainult kahe nivelleerimise kombinatsiooni puhul. Mõlemal juhul oli kaasatud teine või kolmas nivelleerimiskampaania, mille polügoonide sulgemise ajavahemik on samas suurusjärgus või ületab nivelleerimiskampaaniate keskmiste epohhide vahe. Tasandusjärgsete kaaluühiku dispersioonide ja ANOVA-testi põhjal tuvastati keskmiste nivelleerimisepohhide vahel oluline reeperite kiiruste muutus. Kuna aga multivariatiivse analüüsi põhjal tuvastati teise ja kolmanda nivelleerimiskampaania vahel oluline korrelatsioon, jääb selgusetuks, kas reeperite kiiruste muutus on olnud tõeline, füüsikalistel põhjustel toimunud, või juhuslik, üksnes matemaatiline, tuginedes tuvastatud korrelatsioonile. Leitud kiiruste muutust võib põhjendada ka nivelleerimisvigadega. Läbiviidud dispersioonikomponentide hindamine asetas suurema osa nivelleerimisveast esimesele nivelleerimiskampaaniale. Esmakordselt on vertikaalliikumiste määramiseks kasutatud ka Peipsi järve veetaseme andmeid perioodist 1921–2006. Tuginedes viimase kolme nivelleerimise põhjal saadud reeperite vertikaalliikumise kiirustele ja Peipsi järve kallete põhjal leitud veemõõdupostide kiirustele, koostati Eesti maapinna vertikaalliikumiste mudel EST2015LU. Mudeli peamine tunnusjoon on kagu-loodesuunaline jääajajärgne maatõus. Kuid antud mudeli vertikaalliikumiste isobaasid on rohkem lääne-ida suunas kallutatud kui varasemate sama piirkonna kohta koostatud vertikaalliikumise skeemide või mudelite omad. See on tingitud peamiselt neljanda nivelleerimise suuremast mõjust, mida toetavad ka Peipsi veemõõdujaamade kiirused. Saadud vertikaalliikumiste mudeli sobivus GPS-püsijaamade ja ranniku veemõõdujaamade kiirustega oli suurusjärgus ±0,4 kuni ±0,5 mm/a.