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Short-time chirp excitation for wideband identification of dynamic objects

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Abstract. In this paper, some aspects of choosing excitation signals for fast measurement of complex electrical impedance of objects, including impedance of biological tissues, over a wide range of frequencies are discussed. For this purpose, several short-time excitation signals as special cases of chirp signals of near to minimal duration are proposed. The results of analysis and simulation are promising for implementing such kind of signals as the stimulating ones for fast estimation of bioimpedance parameters.

Key words: chirp, titlet, excitation signal, signal energy, spectral density, bioimpedance.

1. INTRODUCTION

Chirps as short-time frequency sweeps are widely used for different purposes. Implementation of chirp signals is common for investigation the parameters and their changes in the objects under study in the acoustic, ultrasonic, optical, seismological, etc. technologies, e.g. [1–4]. Also, applying chirp excitation is known in the biological and biomedical studies, for example, in the bio-impedance measurement, impedance tomography, and in high throughput micro-fluidic devices for analysing the cell cultures [2,5].

Usually, chirps, regardless of their absolutely short length, are considered as signals of many cycles with sinusoidal oscillation. In the simplest cases, the frequency of a chirp rises linearly (so-called up-chirp) or falls in the same manner (down-chirp). Somewhat complicated are the chirps with bidirectional change of instantaneous frequency (double chirps, up-down or down-up). However, chirps with non-linear (quadratic, exponential and more complicated) change of frequency have been applied, as well [3,6]. Furthermore, non-sinusoidal chirps, e.g., signum- or pseudo-chirps are known [7]. Some typical chirp waveforms and their frequency runs are illustrated in Fig. 1.

Bioimpedance can be described by its electrical equivalent (EBI). It is a frequency-dependent complex variable, which has several poles and zeros in different parts of the frequency spectrum inside the comparatively wide bandwidth [8]. Location and shifts of the poles and zeros of the transfer function characterize the state of biological tissues and can give valuable information about the condition of the patient. The common way for EBI measurement is injecting the excitation current with known parameters through the object under test. The comparison of excitation and response signals with following FFT-based spectral analysis enables one to identify the real and imaginary parts of the EBI vector, from which calculation of the magnitude and phase spectra is elementary [8–10]. To get maximum information, some kind of wideband excitation signals like chirps are appropriate. On the other hand, in the development of wearable and implantable medical equipment, low power consumption is important. From this point of view, chirps of minimum length and of the minor power consumption are preferable. Also, shortness of excitation assures high speed of the measurement and helps to exclude impact of changes in a dynamic object during the measurement time.

Below, several modifications of short-time chirps and their parameters will be discussed and compared to typical long chirps. For analysis of different excitation signals an *ad hoc* software simulator was developed. The results were verified in the LabView and MatLab environments.

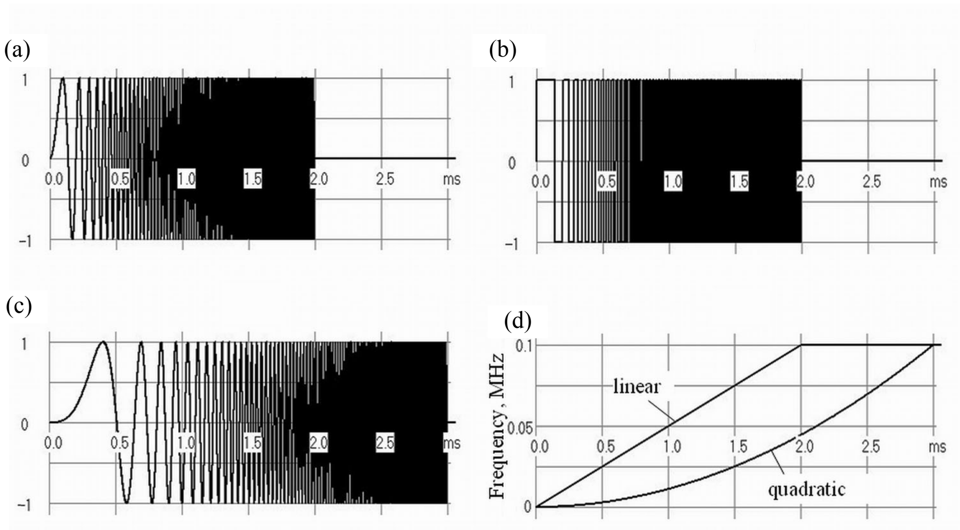

Fig. 1. Normalized waveforms vs time of a 100-cycle chirps with $f_0 = 0$ and $f_{\text{fin}} = 100$ kHz: (a) linear; (b) linear signum-chirp; (c) quadratic; (d) variation (linear and quadratic) of the instantaneous frequency vs time.

2. BASICS OF CHIRP SIGNALS

Generally, a sine-wave chirp signal with instantaneous phase $\theta(t)$ can be described mathematically as $c(t) = \sin(\theta(t)) = \sin(2\pi \int f(t)\mathrm{d}t)$, instantaneous frequency of which changes in time as $f(t) = (\mathrm{d}\theta(t)/\mathrm{d}t)/2\pi$. If the frequency variation $f(t)$ corresponds to some power function of the n-th order, then the change of instantaneous frequency or chirping rate can be expressed as $k_{\mathrm{ch}} = (f_{\mathrm{fin}} - f_0)/T_{\mathrm{ch}}^n$, where f_0 and f_{fin} are the initial and final frequencies, respectively, and T_{ch} is the duration of the chirp pulse. This kind of chirp with the amplitude A is expressed as

$$c(t) = A\sin(2\pi(f_0 t + k_{\mathrm{ch}}\, t^{n+1}/(n+1))). \tag{1}$$

In the case of $n = 1$ we have the linear chirp with constant rate $k_{\mathrm{ch}} = (f_{\mathrm{fin}} - f_0)/T_{\mathrm{ch}}$ (Fig. 1a and d). The rate of an exponential chirp is defined as $k_{\mathrm{ch}} = (f_2/f_1)^{t/T_{\mathrm{ch}}}$, and the chirp of this type can be expressed as

$$c(t) = \sin(2\pi f_0 T_{\mathrm{ch}}(k_{\mathrm{ch}} - 1)/\ln(k_{\mathrm{ch}})). \tag{2}$$

Sometimes chirps with a symmetrical bidirectional frequency change are used (double chirps). In this case the duration of the chirp pulse is $2T_{\mathrm{ch}}$, while the sign of the chirping rate alters at $t = T_{\mathrm{ch}}$.

Selection of the chirp type for practical implementation depends on the task to be solved and often the use of discrete signals is more convenient due to the simplicity of generation and signal processing. The discrete chirps are called signum- or pseudo-chirps with instant values $c_{\mathrm{sgn}}(t) = \mathrm{sign}(c(t))$ (Fig. 1b) [7]. The advantages of signum-chirps are their unity crest factor and major energy comparing with the respective sine-wave chirps of the same length. Unfortunately, the purity of the spectrum is worse. A method to improve the spectrum of the signum-chirp is shortening the duty cycle by certain degrees per a quarter-period [11].

3. SHORT CHIRPS (TITLETS)

Chirp excitation signals need not to be the multi-cycle ones. It appears that single-cycle or even shorter chirps can successfully serve as excitation signals. To specify this kind of very short chirps, the neologism “titlets” was proposed in [12]. The titlets behave similarly to long chirps and thereby have some advantages, e.g, more flat and not fluctuating magnitude spectrum. Of course, at the same frequency bandwidth $B_{\mathrm{exc}} = f_{\mathrm{fin}} - f_0$, short chirps demand substantially higher chirping rate.

Forming of a linear sinusoidal single-cycle chirp is illustrated in Fig. 2 [13]. It is evident that for a chirp of L cycles, the instant phase must be $\theta(t) = 2\pi L$ at $t = T_{\mathrm{ch}}$, where L can be an integer or fractional number of cycles.

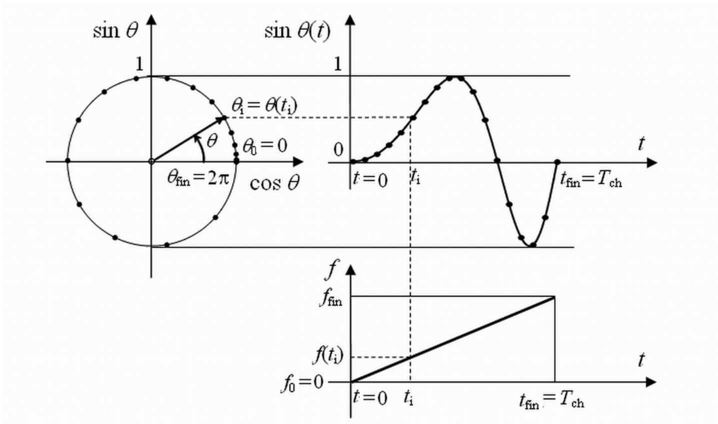

Fig. 2. Waveform genesis of a single-cycle linear sinusoidal chirp at $f_0 = 0$ and $\theta = 0$.

It follows from Eq. (1) that for the n-th order power chirps, the signal length is strongly determined as

$$T_{\text{ch}} = \frac{(n+1)L}{nf_0 + f_{\text{fin}}}. \tag{3}$$

Thus, in the case of a single-cycle linear chirp $(n = 1, L = 1)$ the pulse duration is $T_{\text{ch}} = 2/(f_0 + f_{\text{fin}})$.

Similarly, considering Eq. (2), one can show that for an exponential chirp the duration of a pulse is expressed as

$$T_{\text{ch}} = \frac{L}{f_{\text{fin}}} \ln \frac{f_{\text{fin}}}{f_0}. \tag{4}$$

Some typical waveforms of titlets are shown in Fig. 3. According to Eq. (3), the durations of single-cycle power chirps with $f_0 = 0$ and $f_{\text{fin}} = 100$ kHz are 20, 30 and 40 μs for the linear, quadratic and cubic frequency runs, respectively (Fig. 3a). The duration of single-cycle exponential chirp with $f_0 = 1$ Hz and $f_{\text{fin}} = 100$ kHz is approximately 115.1 μs (Fig. 3b). Figure 3c depicts some waveforms of bidirectional double titlets.

In Fig. 3d, the waveform of a linear single-cycle signum-chirp is shown. The change of polarity for single-cycle sine-wave as well as for signum-chirps occurs at

$$t' = \frac{-f_0 T_{\text{ch}} + \sqrt{f_0^{\,2} T_{\text{ch}}^{\,2} + ((f_{\text{fin}} - f_0) T_{\text{ch}})}}{f_{\text{fin}} - f_0}, \tag{5}$$

which for $f_0 = 0$ gives $t' = \sqrt{2}/f_{\text{fin}}$ (Fig. 3d), while $f(t') = f_{\text{fin}} \sqrt{2}/4$.

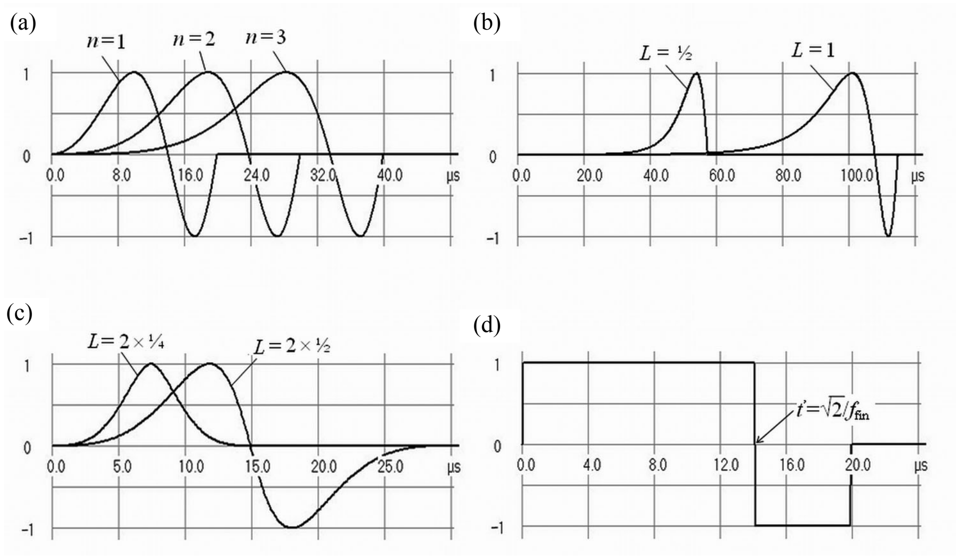

Fig. 3. Waveforms of typical titlets with $f_0 = 0$ ($f_0 = 1$ Hz for exponential chirp) and $f_{\text{fin}} = 100$ kHz: (a) single-cycle sinusoidal power chirps; (b) single- and half-cycle sinusoidal exponential chirps; (c) bidirectional double quarter- and half-cycle sinusoidal quadratic chirps; (d) single-cycle signum-chirp.

Mostly, the titlet pulses have a sizeable DC component, which can be undesirable for some implementations. Knowing the value of t', it is possible to derive the respective compensating value for avoiding the DC. For example, to eliminate the DC component from the single-cycle signum-chirp, the amplitude of the negative half-cycle should be increased to the value of $-(1+\sqrt{2}) \approx -2.414$ V at unity amplitude (1 V) of the positive half-cycle [13].

4. ENERGY OF CHIRPS AND TITLETS

One of the main purposes to use titlets is to minimize the power consumption of generated excitation signals. In general, the total energy of a chirp pulse or titlet $c(t)$ of length T_{ch} can be written in the time domain as $E_{\text{tot}} = \int_0^{T_{\text{ch}}} c(t)^2 \, \mathrm{d}t,$ which leads to $E_{\text{tot}} = (A^2/2)T_{\text{ch}}$ for the long duration sinusoidal waveform. According to the Parseval's theorem, the total energy in the frequency domain must be the same as in the time domain [14]. Unfortunately, for the signals of finite length (i.e., rectangular windowing), a certain part of the signal energy falls outside of the generated excitation bandwidth due to higher frequency components. For that reason, it is necessary to distinguish the total energy E_{tot} of a generated chirp pulse from the useful energy E_{BW}, falling inside the chirp bandwidth B_{exc}. The values of both quantities depend on the pulse length, waveform and spectral nature. We define the energy efficiency as the ratio $\delta_{\mathrm{E}} = E_{\mathrm{BW}} / E_{\mathrm{tot}}$, while mostly $E_{\mathrm{BW}} < E_{\mathrm{tot}}$, and $\delta_{\mathrm{E}} \leq 1$. Theoretically, all the generated energy lies inside the excitation bandwidth ($\delta_{\mathrm{E}} \approx 1$) in the case of a linear long duration sine-wave chirp with $L \to \infty$. For example, the total energy of a sine-wave chirp with $A = 1\,\mathrm{V}$, $f_0 = 0$, $f_{\mathrm{fin}} = 100\,\mathrm{kHz}$, and $L = 1000$ is $E_{\mathrm{tot}} = 9945\,\mathrm{nJ}$ and $\delta_{\mathrm{E}} = 0.998$ (Fig. 4).

For signum-chirps, δ_{E} is smaller even in the case of large L. The problem of the energy efficiency of signum-chirps was theoretically analysed in [15]. The average magnitude spectrum of long signum-chirps decreases gradually outside B_{exc} by frequency intervals of $2 f_{\mathrm{fin}}$ (Fig. 4). It was shown theoretically that for long signum-chirps with $f_0 = 0$, we have $\delta_{\mathrm{E}} = 7\zeta(3)/\pi^2$, where ζ denotes the Riemann zeta function, and $\zeta(3) \cong 1.202$ is known as the Apéry's constant. The latter expression for δ_{E} leads to the rate of energy efficiency about 0.85.

The energy efficiency in frequency domain can be calculated from the results of FFT-processing as

$$\delta_{\mathrm{E}} = \sum_{i=N_0}^{N_{\mathrm{fin}}} \left|V(f_i)\right|^2 \Bigg/ \sum_{i=0}^{N_{\max}-1} \left|V(f_i)\right|^2, \tag{6}$$

where $|V(f_i)|$ is the value of the magnitude spectrum at the i-th frequency bin, N_0 and N_{fin} are the numbers of frequency bins, corresponding to the f_0 and f_{fin}, respectively, and $N_{\max}$ is the total number of frequency bins.

Table 1 presents characteristics of different sine-wave titlets with $A = 1\,\mathrm{V}$ and $f_{\mathrm{fin}} = 100\,\mathrm{kHz}$. The value of P_{av} denotes the average power of a signal, considering the 1 kΩ load.

Table 1 shows that the energy efficiency of titlets is high ($\delta_{\mathrm{E}} > 90\%$), except the quarter-cycle ones. The highest efficiency was measured for the exponential titlets, but their spectral properties are worse comparing to power titlets. We shall deal with this question in the next chapter.

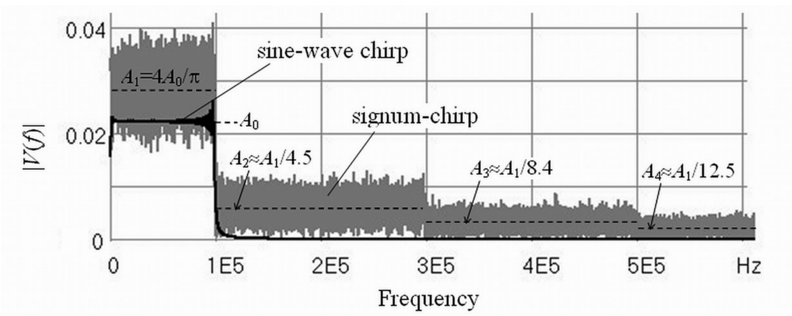

Fig. 4. Magnitude spectra of long linear chirps (L = 1000).

Table 1. Power and energy of titlets

Type of the titlet	L	f_0, Hz	T_{ch}, μs	P_{av}, mW	E_{tot}, nJ	δ_E, %
Linear	¼	0	5.0	0.32	1.6	55.0
	½	0	10.0	0.38	3.8	90.6
	1	0	20.0	0.41	8.3	93.5
Quadratic	¼	0	7.5	0.23	1.8	61.2
	½	0	15.0	0.29	4.4	93.9
	1	0	30.0	0.33	10.0	95.7
Cubic	1	0	40.0	0.28	11.1	96.4
Exponential	1	0.01	161.2	0.097	15.6	97.8
	1	0.1	138.1	0.113	15.6	97.8
	1	1	115.1	0.135	15.6	97.8
Double-quadratic	2 × ¼	0	2 × 7.5	0.23	3.4	93.1

5. SPECTRAL DENSITY OF TITLETS

A basic quantity to characterize spectral properties of a signal is the power spectral density (PSD) function. Considering unity load in ohms, one can calculate the PSD from the FFT-based spectrogram as $\mathrm{PSD}(f) = |V(f)|^2 / B_{\mathrm{FFT}}$, V^2/Hz, where $B_{\mathrm{FFT}} = N_{\max}\Delta f$ is the bandwidth, covered by the FFT processing, $N_{\max}$ is the number of frequency bins, and Δf is the frequency resolution of the transform [14]. The square root $(\mathrm{PSD})^{1/2}$, referred to as the root-mean-square spectral density (RMS-SD), describes the distribution of the signal amplitude in the frequency domain in the units of $\mathrm{V}/\mathrm{Hz}^{1/2}$.

The next diagrams present the RMS-SD for several types of titlets in logarithmic scale, normalized against $[\mathrm{PSD}(f_0)]^{1/2}$. Figure 5 compares RMS spectral density of titlets and long chirps at $f_0 = 0$ and $f_{\mathrm{fin}} = 100$ kHz. Almost ideal rectangular shape of the normalized SD can be achieved at $L >> 1$ (curve 1, Fig. 5a). Diminishing the number of cycles causes intensive fluctuating of the SD, which reduces to a single overshoot at $L = 1$ (Fig. 5b). The aim was to find out

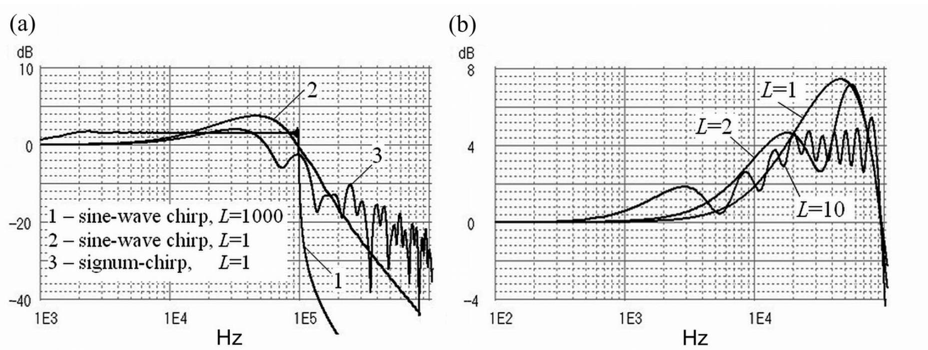

Fig. 5. Comparison of normalized RMS spectra: (a) sine-wave and signum-chirps; (b) sine-wave chirps with different number of cycles.

titlets with improved shape, i.e., ensuring minimal declination of SD inside excitation bandwidth and fast drop-down outside of it.

In Fig. 6, normalized RMS-SD of different titlets are shown. It appears that $L = 1/2$ and $L = 1$ assure better drop-down (–40 dB/dec) than $L = 1/4$ (–20 dB/dec), but in the latter case the SD has less deviation inside the excitation bandwidth (Fig 6a). In fact, the deviation can be further reduced by shortening L below $1/4$, but it is accompanied with drastic loss of the useful energy ($\delta_E < 0.5$).

Generally, the normalized SD of a single-cycle linear titlet does not fall below the zero-dB level inside the excitation bandwidth, but the SD overshoot is quite remarkable (Figs 5 and 6). Somewhat smaller deviation, approximately inside ±5 dB, was achieved using a quadratic titlet (Fig. 6b).

A good flatness of the spectrum within the excitation bandwidth and surprisingly steep drop-down outside it (–80 dB/dec) was observed in the case of the double (bidirectional) quarter-cycle quadratic titlet. Its time-domain and frequency-domain description is shown in Fig. 7. This kind of titlet with f_{fin} = 100 kHz has a total duration of 15 μs and high energy efficiency at the same time (Table 1).

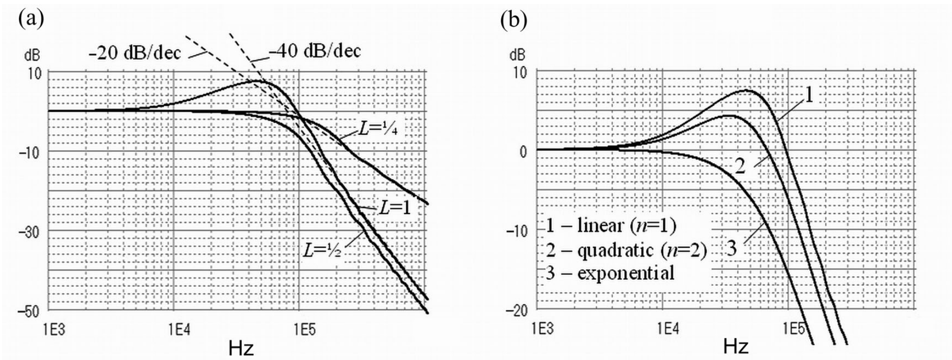

Fig. 6. Normalized RMS spectral density functions of sine-wave titlets with f_{fin} = 100 kHz: (a) linear titlets, n = 1, L = var; (b) titlets of different type, L = 1.

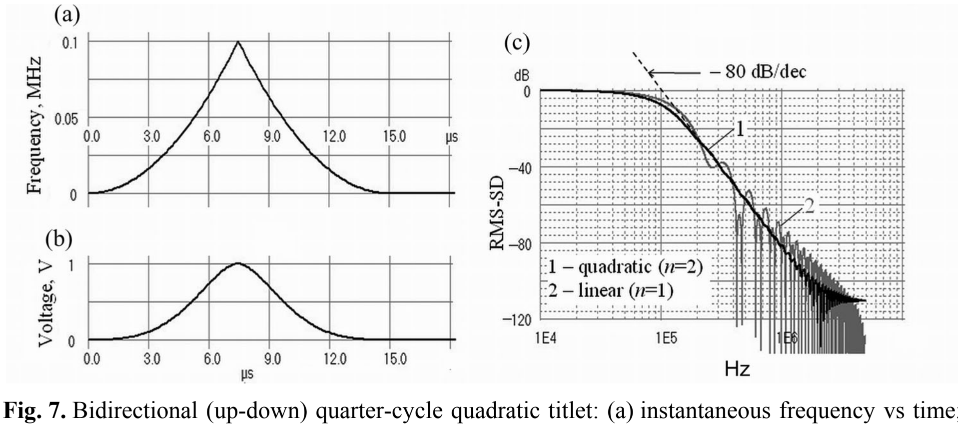

Fig. 7. Bidirectional (up-down) quarter-cycle quadratic titlet: (a) instantaneous frequency vs time; (b) normalized waveform; (c) normalized RMS spectral density.

6. WINDOWING OF TITLETS

In the case of single-cycle sine-wave chirp, the maximum overshoot of normalized RMS-SD is +7.4 dB (at sampling rate of 10 Msamp/s and frequency resolution of $\Delta f = 50\,\mathrm{Hz}$ max $2.56\,\mathrm{mV}/\mathrm{Hz}^{\frac{1}{2}}$ vs $1.09\,\mathrm{mV}/\mathrm{Hz}^{\frac{1}{2}}$ was measured at low frequencies, Figs 5b and 6b). To improve the flatness of the spectrum, a kind of time-domain windowing of chirp pulses should be used [[13]]. As a rule, the windowing is accompanied with some loss of the total energy and power of signals, but usually δ_E still remains quite high. An optimal choice of windows can assure suppressing spectral overshoots and a good drop-down outside the excitation bandwidth.

In the frame of this research, several typical [[16]] and specific windows were studied. Additionally, the squared-sine window as a particular case of the Tukey window with function $w(t) = \sin^2(\pi t/T_\mathrm{ch})$ and non-symmetric windowing were used. The latter is specified by the function $w(t) = (t/T_\mathrm{ch})^a$. Some example waveforms and respective spectral densities in a magnified scale are presented in Fig. 8.

In Fig. 8a, the impact of windowing to the bidirectional quarter-cycle titlet is shown. The best flatness was achieved by the Nutall window, use of which reduces the deviation down to 2 dB inside the excitation bandwidth. Somewhat worse, but acceptable, was the impact of the Hanning, the Hamming and Tukey windows.

Figure 8b shows the effect of non-symmetrical windowing on the single-cycle linear titlet. In this case the deviation of RMS-SD remains inside the boundaries of ±1 dB at α = 2.5.

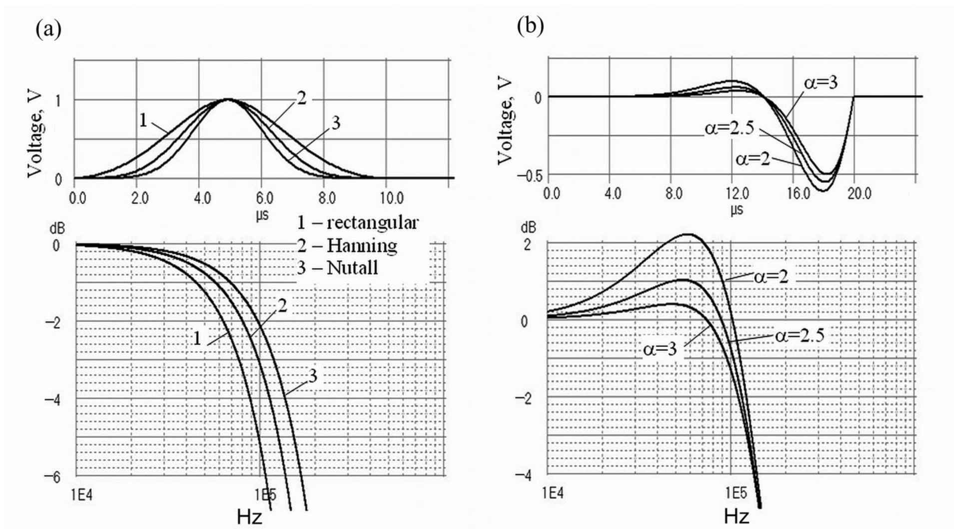

Fig. 8. Waveforms and RMS-SD of windowed titlets: (a) double quarter-cycle (L = ¼) linear titlet; (b) single-cycle (L = 1) linear titlet, non-symmetric windowing, α = var.

7. CONCLUSIONS

Short-time chirps (known also as titlets or minimum-length chirps) preserve the unique properties (wide bandwidth and flat spectrum) of typical chirps. For several types of titlets, the flat spectral density of excitation is observed from very low frequencies (even from DC) up to tens of MHz range, depending on the chirping rate, which can be very high. Moreover, the flatness of spectral density can be improved by using appropriate time-domain windowing of the generated signal.

Short duration of titlets involves low energy consumption, necessary for generating excitation signals in the case of limited capabilities of the power supply. An important quantity of titlets is the energy efficiency, i.e., the amount of useful energy, which falls into the excitation frequency band B_{exc}. The energy efficiency of linear sine-wave chirps exceeds usually 90%, and is more than 80% for signum-chirps. All these properties assure the opportunity for fast and wide-band observation and identification of processes together with minor power consumption in dynamic objects with high-speed changes, so making the titlets promising candidates for implementation in wideband spectroscopy of dynamic impedances, and in other applications, where the broadband high-speed measurement together with low energy consumption is required.

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Lühikesed sirinsignaalid dünaamiliste objektide laiaribaliseks identifitseerimiseks

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On analüüsitud väga lühikeste sirinsignaalide (*chirp signals*) kasutusvõimalusi ergutussignaalidena erinevate dünaamiliste objektide, eelkõige bioloogiliste objektide impedantsi mõõtmisel. Signaali lühiduse all on siinjuures silmas peetud tsüklite arvu, st on vaadeldud sirinsignaale, mille kestus on üks tsükkel või vähem ja mille hetkefaasi muutus on seetõttu maksimaalselt 2π.

Kui sirinsignaalide amplituudispekter on määratud nende signaalide alg- ja lõppsagedusega ning on üldreeglina lai ja küllalt ühtlane kogu spektri ulatuses, siis lühidus tagab täiendavalt väikese energiatarbe signaalide genereerimisel, mis on väga oluline biomeditsiinitehnikas kasutatavates portatiivsetes ning implanteeritavates rakendustes.

Artiklis on käsitletud nii lineaarseid kui ka mittelineaarseid sirinsignaale, avaldatud nende ajalise kestuse sõltuvus tsüklite arvust ja piirsagedusest ning kirjeldatud nende signaalide energeetilist kasutegurit. Illustratsioonidena on toodud näiteid signaalide kujust aja- ja sagedusvallas spektraaltiheduse funktsioonidena. Lisaks on vaadeldud võimalusi amplituudispektri kuju korrigeerimiseks.